

Index

- Abel integral equation, 207
 accumulation, point of, 9
 acoustic source, 421
 almost everywhere, 28
 anisotropic medium, 429
 anti-dual, 19
 anti-symmetry about plane, 268
 asymptotic development, 345
 asymptotic series, 345
 asymptotics
 of elementary solution, 427–9
 of finite integral, 364–78;
 non-uniform, 370
 of inverse polynomials, 386–9
- Banach space, 16
 base, countable, 12
 Bessel function
 $H_0^{(2)}$, 415, 416
 I_ν , 483, 484
 J_ν , 112, 113, 154, 159, 193–5, 207,
 208, 249, 250, 256, 259, 284, 335,
 348, 349, 351, 364, 384, 397, 403,
 419, 483, 505, 508.
 K_ν , 178–80, 182, 259, 260, 335, 339,
 413, 418
 Y_ν , 179, 397, 403
 Beta-function, 113, 114, 202
 biharmonic equation, 420, 421
 Bolzano–Weierstrass property, 9
 bound
 greatest lower, 4
 least upper, 4
 boundary value problem, 410, 411
- Cauchy principal value, 34, 102
 Cauchy sequence, 13
 Change of variable
 in convolution, 286
 in integral, 285
 linear, 271, 487
 periodic, 300–7
 on R_1 , 273–86, 485–7
 on R_n , 287–92, 487, 488
 transform of, 272
- characteristic boundary value, 411
 class, 1
 C_m , 91
 finite, 3
 closure, linear, 15
 coherence theory, 405
 communication theory, 405
 comparison test, 26
 complement of set, 2
 composition, 176
 conical point, 378, 381, 384
 continuity
 absolute, 460
 uniform, 25
 continuous function theorem, 57
 convergence
 absolute, 26
 of directed set, 11
 pointwise, 22, 27
 of sequence, 4
 of series, 25
 uniform, 22, 27
 convolution
 associativity of, 182, 194, 199, 201,
 208, 341, 344
 change of variable in, 286
 derivative of, 183, 186
 fairly good function in, 176–8, 258
 Fourier transform of, 181, 187–97,
 205, 259
 in K_+ , 200
 in K_+ , 340
 of weak functions, 451–8;
 associativity of, 453, 456
 covariance, 429
 covering, open 10
 curvature, Gaussian, 372
 cylindrical polar coordinates, 211
- $\delta(x)$
 approximating sequence, 78, 79
 definition, 55
 Fourier transform of, 72
 integral of, 168
 multiplication by, 167, 450

- $\delta(x)$
 in curvilinear coordinates, 291, 292,
 304, 306, 307
 definition, 216
 as direct product, 236
 as limit, 277, 228, 257, 314
 multiplication by, 258, 482
 on R_2 , 246
 $\delta(\omega \cdot x)$, 266
 $\delta(\sin x)$, 285
 $\delta(f)$, 292–300
 $\delta(q)$, 296–300
 $\delta(q)H(x_1)$, 328, 329
 derivative, 28
 with change of variable, 275
 of convolution, 183, 186
 cross, 31
 fractional, 204, 343
 of generalised function, 66
 derived number, 28
 diameter of set, 12
 diffeomorphism, 287
 difference between sets, 2
 differential-difference equation, 508
 differential equation, ordinary, 405–10,
 460–72
 elementary solution of, 407, 427, 466
 Laplace transform of, 506–8
 operational method, 467–72
 properties of, 407–9
 solution of, 460, 461
 in W_+ , 463
 differential equation, partial, 410–27
 Laplace transform for, 513–15
 differential equations, systems of, 409
 operational solution, 472–78
 partial, 421–7
 differential operator of second order, 223
 direction, 8
 disjoint class, 2
 distance between sets, 13
 distance function, 12
 distributions, 58, 495–7
 divergent series, 25
 division of generalised functions, 98,
 112, 171–6, 239
 domain, 5
 duality, principle of, 3
 elastic waves, 429
 electromagnetic waves in crystal, 428
 electron, radiation from, 421, 424
 elementary solution
 in anisotropic medium, 429
 asymptotic behaviour, 427–9
 of biharmonic equation, 420, 421
 of harmonic telegraph equation, 414,
 418
 of Helmholtz equation, 416, 421
 of Laplace's equation, 412, 420
 of Maxwell's equations, 422–4
 of ordinary differential equation, 407
 of partial differential equation, 411,
 426, 427
 of Stokes flow, 425
 of supersonic flow, 429
 of telegraph equation, 419
 of ultrahyperbolic equation, 418
 of wave equation, 415, 419
 entire function, 489
 equivalence class, 3
 equivalence relation, 3
 extension, 18
 of continuous function, 27
 factorial function, 107
 logarithmic derivative, 114, 119, 121
 Faltung, 176
 Fatou's lemma, 141
 fine function, 46, 212
 $\rho(x)$, 59
 $\sigma(x)$, 46
 $\tau(x)$, 46
 Fourier transform of, 489
 Fourier inversion theorem, for functions,
 48
 for generalised functions, 73; on R_n ,
 224
 for good functions, 51; on R_n , 214
 for ultradistributions, 492
 for weak functions, 492
 Fourier series, 144–55
 uniqueness, 147
 for periodic generalised function, 151
 Fourier transform
 complex, 491
 of convolution, 181, 187–97, 205, 259,
 343
 of direct product, 237
 of fine function, 489
 of generalised function, 71–5
 of general metric, 339
 of good function, 46–52
 of hyperbolic distance, 329–37
 in L_2 , 143
 in L_2 , 260
 of parametric integral, 266
 of periodic generalised function, 301

- of power of r , 251–7
- of power of x , 109–113
- as repeated integral, 235
- in R_n , 212, 224–6, 251
- of series, 128
- on subspace of R_n , 234
- table, 529–32
- of ultradistribution, 492
- of ultrahyperbolic distance, 329–37
- of weak function, 491
- function, 5
 - bounded, 20
 - of bounded variation, 30
 - continuous, 8, 11, 57
 - discontinuous, 21
 - entire, 489
 - fairly good, 45, 65; on R_n , 212
 - fine, 46; on R_n , 212
 - as generalised function, 58–62, 217–19
 - good, 44; on R_n , 211
 - integrable, 32
 - inverse, 6
 - moderately good, 45
 - monotone, 29
 - numerical, 19
 - of positive type, 429, 430
 - step, 32
 - uniformly continuous, 25
 - as weak function, 435
- functional, linear, 17, 58
 - bounded, 17; continuous 17, 58, 496
- Gaussian curvature, 372
- general convergence, 128–32
 - of Fourier series, 146
- generalised function, defined by fine functions, 181
 - definition, 54
 - derivative of, 66–71
 - even, 97–9
 - on finite interval, 277
 - Fourier transform of, 71–5, 90; on R_n , 224–8
 - on half-line, 121–7
 - in Hermite polynomials, 137–41
 - homogeneous, 271–3
 - on hypersurface, 240
 - indefinite integral of, 85
 - infinite integral of, 168, 170
 - with isolated singularities, 349, 363; and periodic, 355
 - odd, 97–9
 - parametric derivative of, 80
 - partial derivative of, 220–3
 - periodic, 149–55, 160, 161, 355
 - positive, 92, 93
 - of positive type, 431
 - on R_n , 216
 - structure of, 84; on R_n , 229
 - as weak function, 435
 - well behaved, 348
 - zero except on manifold, 289
 - zero except at origin, 89
- generalised functions, bounded class of, 83
 - multiplication of, 162–72
 - sum of, 63; on R_n , 218
 - trigonometric, 155
- good function, 44, 211
 - in Hermite polynomials, 135
- Green's function, 466 (*see also* elementary solution)
- Green's theorem, 479
- Hadamard finite part, 101, 102
- Hahn–Banach theorem, 18
- Hartmann number, 426
- Heaviside unit function, 63
 - derivative of, 68
 - Fourier transform of, 112
- Heine–Borel property, 10
- Helmholtz equation, 416, 421
- Hermite polynomials, 132–41, 394
 - associated orthonormal set, 134–41, 159, 232, 394
- Hilbert space, 16
- Hilbert transform, 193, 396–405
- Hölder's inequality, 35, 43
- Huygens principle, 417
- hyperbolic distance, 310–37
 - Fourier transform of, 329–37
- hypergeometric function, 256
- image, 6; inverse, 6
- inclusion, 1
- infimum, 4
- initial value problem, 416, 466
- integral
 - absolutely convergent, 35, 39
 - Cauchy principal value, 34, 102
 - change of variable in, 285
 - fractional, 204, 207, 343, 457, 483, 484
 - of generalised function, 85, 168, 170, 207; on R_n , 230
 - infinite, 34
 - iterated, 43
 - Lebesgue, 33

- parametric, 263–6, 484, 485
 on R_n , 211
 Riemann, 32; double, 42
 singular, 99, 124, 167–70, 308
 Stieltjes, 33, 91
 uniformly convergent, 38
 of weak function, 445
 integral equation, 192–5, 206–8, 343,
 344, 390–403, 519, 520
 homogeneous, 390
 Volterra, 390, 395, 396, 458–60, 509,
 510
 Wiener–Hopf, 520–4
 integro-differential equation, 390
 intersection
 of sets, 2
 of classes, 2

 J_ν , *see* Bessel function
 Jones, D. S., 379

 Kelvin's stationary phase, 357
 kernel, 390
 Kirchhoff's formula, 420
 Kline, M., 379
 König, H., 90
 K_p – space, 58
 K_+ – space, 200; Fourier transform
 in, 204
 K_p^p – space, 217
 K_+^p – space, 340; Fourier transform
 in, 341

 L_p , 35, 58
 L_p' , 184
 L_{∞}^p , 399
 L^m , 488
 L_p , 217
 L_p^p , 260
 L^p , 482
 $\mathcal{L}_+(c)$, $\mathcal{L}'_+(c)$, 500
 $\mathcal{L}_1(c)$, $\mathcal{L}'_1(c)$, 510
 $\mathcal{L}_2(c_1; c_2)$, 510
 $\mathcal{L}(c_1, c_2)$, $\mathcal{L}'(c_1, c_2)$, 516
 Laplace transform, 500–28
 bilateral, 516–19
 of convolution, 505
 inversion of, 503
 on $\mathcal{L}'_+(c)$, 500
 on R_n , 510–13
 regularity of, 502
 table, 533
 Laplace's equation, 263, 412, 413, 420
 Laplacian of generalised function, 223

 Lebesgue–Stieltjes measure, 29
 Leibnitz's rule, 448
 Lerch's theorem, 521
 Levi's theorem, 141
 Liénard–Wiechert potential, 421, 424
 Lighthill, M.J., xi, 44
 limit
 of directed set, 8
 double, 23
 generalised, 75–81; completeness of,
 130; on R_n , 226–8
 iterated, 23
 in metric space, 21–5
 weak, 438; completeness of, 443, 447
 limit inferior, 4, 5
 limit point, 9
 limit superior, 3, 5
 Liouville's theorem, 523
 logarithm, 98
 Fourier transform of, 117–21, 252,
 254–7
 of hyperbolic distance, 325–8, 333
 with power, 115, 253, 325

 magnetohydrodynamic waves, 429
 Maxwell's equations, 422–4
 mean value theorem
 first, 37, 43
 second, 41
 measure, 29, 31
 measure zero, 28, 31
 metric, 12
 Minkowski's inequality, 36, 43
 multiplication of generalised functions,
 162–72
 associativity of, 164
 on R_n , 257
 multiplication of weak functions,
 447–51, 482

 neighbourhood, 7
 punctured, 7
 numbers
 extended real, 5
 infinite, 5
 monotone sequence of, 5
 real, 4

 open set in metric space, 13
 operational method, 467–72
 for system, 472–6
 operational representation, 468
 ordering, partial, 8
 orthogonal mapping, 267

- partial derivative of generalised function, 220
 commutativity of, 221
 in volume integral, 221–3
- partial fraction, 112, 175, 176
- point of accumulation, 9
- Poisson summation formula, 155–60, 307
- Poisson's formula, 419
- powers of x
 definition of, 94–7, 104–7
 Fourier transform of, 109–12
 with logarithm, 115–21
- powers of r , 246–9
 Fourier transform of, 251–7
- powers
 of quadratic metric, 337–9
 of hyperbolic distance, 310–29
 of ultrahyperbolic distance, 310–29
- product, direct, 235–42
 associativity of, 236
 derivative of, 236
 Fourier transform of, 237
 of weak functions, 481
- product, with fairly good function, 65, 165, 219
 infinite, 172
 scalar, 16
- product set, 6
- product space, 6
- $\psi(x)$, 114, 119, 121
- R_1 , 5
- R_n , 20
- random process, 429
- range, 6
- ratio test, 26
- real line, 5
- restriction, 18
- resultant, 176
- Reynolds number, 426
- Riemann–Lebesgue lemma, 345
- Riesz representation theorem, 34
- rotation of axes, 267; invariance under, 268
- Schwartz, L., 58, 495, 497
- Schwarz inequality, 16, 35, 43
- sequence, 3
 Cauchy, 13
 convergence of, 4
 equicontinuous, 87, 230
 monotone, 4
 non-decreasing, 4
 non-increasing, 4
 regular, 53, 216, 434; equivalent, 53, 216, 434
- series, 25–8
 general convergence of, 128–32
 uniformly convergent, 40, 43
- set, 1
 closed, 7, 20
 complement of, 2
 countable, 3
 dense, 12
 denumerable, 3
 directed, 8
 empty, 1
 finite, 3
 index, 2
 interior of, 7
 linearly ordered, 8
 of measure zero, 28, 31
 open, 7, 20
 product, 6
- sets
 disjoint, 2
 laws for, 2
- signal
 real, 405
 analytic, 405
- signature, 374
- signum, 63
- Silva, J.S.e., 90
- singular integrals, 99, 124, 167–70, 444;
 on R_n , 308
- Sonine's integral, 208
- Space, 1
 anti-dual, 19
 Banach, 16–18
 compact, 9–12
 complete, 13
 dual, 18, 58
 function, 15–19
 Hausdorff, 8
 Hilbert, 16
 linear, 15
 linear metric, 15
 locally compact, 11
 metric, 12–14
 normed linear, 15
 product, 6
 reflexive, 19
 separated, 8
 of sets, 1
 topological, 7
- sphere, 12
- spherical polar coordinates, 226

- stationary phase
 on R_1 , 357–62
 on R_n , 379–86
stationary random process, 429
Stieltjes integral, 33, 91
Stokes flow, 425, 426
subsequence, 4
subspace, linear, 15
sum
 of classes, 2
 of sets, 2
supersonic flow, 428
supremum, 4
symmetry about plane, 268
- Taylor series, 37, 485, 494
telegraph equation, 419
 harmonic, 414, 418
Temple, G., xi
Titchmarsh, E. C., 186, 456
Titchmarsh's theorem, 497–9
topological space, 7
topological equivalence, 8
topological, 7; induced, 7
triangle property, 12
trigonometric generalised function, 155
- ultradistributions, 488–95
ultrahyperbolic distance, 310–37
union
 of classes, 2
 of sets, 2
- variety, linear, 15, 18
Volterra integral equation, 390, 395,
 396, 458–60
 by Laplace transform, 509, 510
Volterra's formula, 419
- W_+ , 454
 \mathcal{W}_+ , 482
Watson, G. N., 178, 259
wave equation, 412, 416–19, 513–15
 harmonic, 415, 421, 524–8
 Kirchhoff's solution, 420
 Poisson's solution, 419
 Volterra's solution, 419
weak function, 434
 derivative of, 437
 as distribution, 496
 even, 444
 integral of, 445
 positive, 443
 on R_n , 478–84; structure of, 480
 structure on R_1 , 439–43
 zero except at origin, 443
weak functions, series of, 444
weak limit, 438
 completeness of, 443, 447
Weierstrass's M -test, 27
Weierstrass's theorem, 480
Wiener–Hopf technique, 520–8
- Zorn's theorem, 18