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INTEGRAL EQUATIONS

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PREFACE

The present work is intended as a successor to Maxime Bôcher's tract *An introduction to the study of integral equations*, which has long been out of print. It is devoted entirely to non-singular linear integral equations, that is, those for which the main results of the Fredholm theory are valid. Only a brief indication of the important applications to differential equations is given, in § 1·2 of the Introduction.

The theory is not presented in terms of linear operators in Hilbert space or a more general topological vector space; this has enabled me to obtain stronger results on the convergence of the various expansions than would have been possible in a more general context. On the other hand, I have made extensive use of the notations of operator theory; many of the formulae thus become much briefer and easier to read.

The Lebesgue integral is used throughout. Most of the theory is given for the case of $\mathfrak{L}^2$ kernels (defined in § 1·6), which illustrate most of the phenomena likely to be encountered; the definition is framed in such a way as to make equations hold everywhere instead of almost everywhere, whenever it is possible to do so. Much information about more general kernels is to be found in A. C. Zaanen's *Linear Analysis* (1953). I have made systematic use of E. H. Moore's relatively uniform convergence (§ 2·4), which seems to fit into the theory in a remarkably natural way.

The literature of the subject is now enormous, and I make no claim to completeness for the historical remarks or for the bibliography; I have given references for the classical results and some of the less familiar ones, and for results not belonging to the theory itself. References in the text to the bibliography are given thus: Fredholm (1903), Schmidt (1907*a*).

My thanks are due to the late G. H. Hardy, who inspired my early work in the field and first suggested the present enterprise, to my wife for her constant encouragement and for help in preparing the index, to Mr A. L. Brown for assistance in correcting the proofs, and to the Cambridge University Press for the care and accuracy of their printing.

F. S.

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NOTE ON THE SECOND IMPRESSION

Almost all the changes in this reprint are corrections of minor errors. My thanks are due to those readers who have made suggestions for corrections and improvements, especially to Professor I. A. Barnett, who provided a most detailed and careful list.

Two excellent books containing much material beyond the scope of this tract have appeared since the bibliography was prepared. These are:

S. G. MIKHLIN (1957). *Integral equations and their applications to certain problems in mechanics, mathematical physics and technology*. Translated by A. H. Armstrong. London, New York, Paris, Los Angeles.

F. G. TRICOMI (1957). *Integral equations*. New York, London.

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