

Cambridge University Press

978-0-521-09871-7 - Cartesian Geometry of the Plane

E. M. Hartley

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# CARTESIAN GEOMETRY OF THE PLANE

BY  
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CAMBRIDGE  
AT THE UNIVERSITY PRESS

1966

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CAMBRIDGE UNIVERSITY PRESS  
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi

Cambridge University Press  
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

[www.cambridge.org](http://www.cambridge.org)  
Information on this title: [www.cambridge.org/9780521052221](http://www.cambridge.org/9780521052221)

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First published 1960  
Reprinted 1966  
This digitally printed version 2008

*A catalogue record for this publication is available from the British Library*

ISBN 978-0-521-05222-1 hardback  
ISBN 978-0-521-09871-7 paperback

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## PREFACE

This book is intended as a first course in coordinate geometry, of the type envisaged in the Report on the Teaching of Higher Geometry in Schools, prepared for the Mathematical Association in 1953. I have had in mind those working for the Advanced and Scholarship levels of the General Certificate of Education, but there is sufficient material to meet also the needs of those reading for a General degree including Mathematics whose requirements stop short of the use of homogeneous coordinates. I have, as the title suggests, confined my attention to the Cartesian plane, feeling that most students benefit from gaining certainty and confidence in this field. Those who later specialize in Mathematics should be in a position to appreciate other geometries, after thoroughly mastering one, while those whose interest lies elsewhere will have certain necessary tools in their hands, and experience in using them.

I have tried to combine clarity in the exposition with more rigour than seems to be usual at this stage. It is probably unwise to insist on absolute standards of rigour from a pupil so early in a mathematical career, but the demands made later for logical thought will seem less unreasonable to the student who is accustomed to knowing exactly what has been proved. Although the chief emphasis is on the method of coordinates, I have not hesitated to use a combination of pure and analytical methods at every stage. References are made to other branches of mathematics, and to some methods of drawing the curves considered, which I hope will make the subject seem relevant and interesting.

For those working with little or no help I have given in the Introduction a summary of those results in Algebra, Pure Geometry, Trigonometry and Calculus which are used in the sequel. Proofs of the theorems in the Geometry section of this Introduction are outlined; for the other parts of the work the reader should consult the appropriate text-book. Some paragraphs are in smaller type; these contain comments which are included in the hope of answering some of the questions which may arise in the mind of the reader. These sections, and paragraphs marked with a star, may be omitted (and during a first

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reading probably should be), without the thread of the argument being lost.

The examples in each chapter are meant to provide immediate illustration of, and practice in the use of, the formulae (emphasized in bold type) which have just been obtained. Enough of these should be worked to make each formula thoroughly familiar; it is particularly important for those working alone not to omit such routine 'drill'. At the end of each chapter there is a set of miscellaneous examples which require more thought; the harder of these appear below a line. Some further examples, covering topics treated in different chapters, appear after the final chapter. In some of the cases where an example is not a straightforward application of book-work, a hint as to an appropriate method of attack is given at the end of the book.

The number of those who have helped in different ways to make the book a reality is legion. In particular I must mention Dr E. A. Maxwell, of Queens' College, Cambridge, from whom I learned much as an undergraduate; he first suggested my writing on this subject, and with astonishing generosity read and criticized much of the manuscript. In this exercise he was later joined by Mr A. P. Rollett and Dr H. M. Cundy, and to each I owe gratitude for many useful suggestions. Among former pupils who gave me advice founded on their experience in the class-room, I am specially indebted to Miss E. A. Dickens, who also, with Miss I. L. Campbell and Miss P. E. Moss, provided the final list of answers. Many of the examples are taken from past papers of various examining bodies, and I wish to acknowledge my indebtedness to the University of Bristol; the University of Cambridge; the University of Sheffield; Girton College, Cambridge; Newnham College, Cambridge; Royal Holloway College, London; St Hugh's College, Oxford; Somerville College, Oxford; Cambridge Local Examinations Syndicate; Northern Universities Joint Matriculation Board; Oxford Local Examinations; Oxford and Cambridge Schools Examination Board; University of London Examinations Board for permission to use these questions from their papers. Finally, I am grateful for much encouragement and tolerance shown by the staff of the Cambridge University Press to a newcomer to the art of producing books.



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Throughout, I have been aware of my indebtedness to a friend and former colleague, the late Dr Christine Hamill, of Newnham College, Cambridge, the University of Sheffield, and University College, Ibadan. I remember with pleasure and gratitude the many occasions on which together we evolved our methods of teaching, and in particular a few days' holiday in September 1955, during which we discussed the form and contents of this book; it was she who, with recent experience of the needs of West Africa in mind, urged me to remember those who would have to work largely alone. From these conversations, and from a subsequent six months correspondence, during which she criticized the first few chapters in merciless detail, I gained untold stimulation. I have tried to capture and communicate the interest, even fascination, of geometry for us; if in any measure I have succeeded, the praise is hers.

E. M. H.

CAMBRIDGE  
*August 1959*