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P. C. Clemmow

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THEORY**

**P. C. CLEMMOW**

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## PREFACE

Electromagnetism is central to so many aspects of physics, engineering and applied mathematics that it must form a significant part of the curriculum of each of these disciplines. On the other hand the continual broadening of the scope of these disciplines as knowledge advances means that progressively less time can be devoted to the study of their more traditional parts: and whilst electromagnetism is very much alive, and itself contributes to the demand for the admittance of new topics into educational courses, there is a need to put across its fundamentals more crisply than has been the fashion in the past.

This book aims to give reasonable coverage of first and, depending on the syllabus, some second and possibly third-year university work in an introductory text that is comparatively short but fully explanatory. Some sacrifice was required, and it may be helpful to indicate where the axe has fallen.

The main omissions arise in two ways. First, the account is confined to theory, and that mostly field theory; it does not include to any significant extent descriptions of experimental phenomena or technical applications. Secondly, the treatment assumes a rather specific mathematical competence on the part of the reader; namely, that he is conversant with the standard results of vector algebra and vector calculus summarized in the Appendix.

Which said, it must be emphasized that the treatment is in no sense at a sophisticated theoretical level aloof from practicalities. On the contrary, it aims to foster an awareness of orders of magnitude in practical units; and whilst omitting any discussion of the mathematics of vector fields *per se*, which so often takes up space in texts on electricity and magnetism, it also omits material that requires other significant mathematical expertise. Thus Legendre polynomials, Bessel functions and the like are kept out, in the belief that for illustrative purposes there are enough problems soluble in terms of elementary functions: the only concession is the brief appearance of elliptic integrals in the discussion of inductance. Furthermore, the commitment to standard vector analysis is total. The temptation to introduce cartesian tensors was strong, and resisting it played some part in prescribing the coverage; in particular, by effectively curtailing discussions of electromagnetic momentum and of anisotropic media, which are best conducted in tensor terms.

The topics covered are by and large conventional, as a glance at the Contents shows. The foundations of electromagnetic theory were laid over a century ago, and must be mastered before applications, whether old or new,



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## PREFACE

can be appreciated: even Feynman, with reference to the second year of his *Lectures on Physics*, wrote ‘In the first part of the course, dealing with electricity and magnetism, I couldn’t think of any really unique or different way of doing it – any way that would be particularly more exciting than the usual way of presenting it.’ But this is not to say that yet another introductory text cannot offer something worthwhile. As Bob Hope observed, when asked what Dorothy Lamour had that others hadn’t, ‘Nothing, but she groups it better’.

In the present book a comparatively unusual feature of the way the topics are grouped is the early development of Maxwell’s equations. The traditional route that leads to them is taken in Chapter 2, without the prior *detailed* treatments of electrostatics, magnetostatics and induction that so easily eat up the pages, and whose many and varied individual aspects may divert attention from the fundamental unity of the theory. Time independent, and slowly varying situations are considered in Chapters 3 and 4, of which perhaps all that need be noted here is the introduction of dielectric and magnetic media on a purely macroscopic, phenomenological basis. Chapter 5 deals with electromagnetic waves, covering important aspects that do not need to invoke mathematics beyond the limits already indicated. and again considering media in the simplest phenomenological way.

Finally, Chapter 6 introduces media in essentially classical microscopic terms. The dipole moment densities  $\mathbf{P}$  and  $\mathbf{M}$  make their first appearance, and a number of elementary models are investigated. The theory is necessarily less clear cut than that in previous chapters, partly because of the inherent difficulty of attempting to describe classically what may well be truly quantum effects. There is also the complication of defining an ‘average’ field at points inside a molecular medium. This is one of those questions that can be a perennial thorn in the flesh of an honest teacher. Varied treatments in the literature seem insufficiently incisive, or at least too complicated, to include in an elementary book: yet to gloss over the matter is likely to lead to confusion. Opportunely, however, a comparatively straightforward procedure has recently been proposed,<sup>†</sup> and the gist of this is incorporated in §6.1.3.

Some problem solving is an essential part of learning, and a set of problems has been provided which is quite closely related to the text; hopefully, their solution demands an understanding of the material rather than, save here and there, mathematical dexterity or perseverance.

It is rare to feel of a text book ‘that’s how I would have done it’, and it would be rash to suppose that any of my colleagues will so react to this one. Nevertheless, I recognize gratefully that many of them, both at Cambridge and elsewhere, have influenced the outcome. To them my thanks is col-

<sup>†</sup> A derivation of the macroscopic Maxwell equations, G. Russakoff (1970), *American Journal of Physics* **38** (10), 1188–95.

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lectively and sincerely offered. Specifically I am indebted to Professor Colin Hines for the quoted reference and forceful advocacy, of its viewpoint which he had arrived at independently. Thanks are also due to the University Press for their expertise in dealing with the typescript and great patience in awaiting its arrival. Responsibility for the delay was entirely mine, and it could have been longer but for the efficient typing of Miss Hilary Holden and Miss Susan Whitelegg.

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P. C. C.