

## INDEX

- addition of homomorphisms, 38  
 additive functor, 143  
 adjacent primary submodules, 202  
 affine subset, 379  
 algebraically closed field, 285  
 algebraic element with respect to a field, 283  
 ambient set, 78  
 annihilator of a module, 63  
 anti-isomorphism of a ring, 40  
 Artin–Rees Theorem, 210, 292  
 Artin ring, 181  
 ascending chain condition for submodules, 22  
 associative law for multiplicities, 342  
  
 base for a topology, 378  
 base of a free module, 59  
 boundary homomorphisms of a complex, 351  
  
 canonical filtration on a completion, 390  
 canonical homomorphism into a completion, 390  
 canonical mapping of a module into a module of fractions, 131  
 Cartesian product, 2  
 Cauchy sequence, 386  
 central element of a ring, 68, 290  
 central ideal, 290  
 centre of a ring, 290  
 chain factors, 18  
 chain of submodules, 18  
 closed set, 377  
 closure of a subset of a topological space, 378  
 Cohen’s Theorem, 231  
 comaximal ideals, 188  
 commutative diagram, 56  
 commutative ring, 1  
 compatibility of a filtration on a module with a filtration on a ring, 404  
 compatible homomorphisms of filtered modules, 383  
 complement of a subset, 75  
 complete filtered module, 386  
 completion of a filtered module, 389  
 complex of modules, 351  
 component of a submodule determined by a multiplicatively closed set, 103  
 composition factors, 19  
 composition series, 19  
 congruence modulo a submodule, 12  
 connecting homomorphism, 354  
 continuous mapping, 379  
 contraction of an ideal in a subring, 86  
 contraction of a submodule of a module of fractions, 137  
 contravariant functor, 142  
 convergent sequence, 385  
 convergent series, 387  
 cosets of a submodule, 12  
 covariant functor, 141  
 cumulative Hilbert function, 317  
  
 decomposable ideal, 102  
 decomposable submodule, 102  
 degree formula for polynomials, 267  
 degree of a homomorphism of graded modules, 117  
 degree of a polynomial, 266  
 descending chain condition for submodules, 23  
 differentiation homomorphisms of a complex, 351  
 dimension of a (commutative) ring, 221  
 dimension of a module, 249  
 dimension of a prime ideal, 221  
 dimension of a proper ideal, 221  
 dimension of a vector space, 167  
 direct summand, 142  
 direct sum of modules, 31  
 direct sum of rings, 41  
 division ring, 39  
  
 embedded prime ideal, 106  
 embedding, 4  
 endomorphism of a module, 39  
 epimorphism, 5  
 equivalent chains, 18  
 Euler–Poincaré characteristic of a complex, 369  
 everywhere dense set, 378  
 exact functor, 145  
 exact homology sequence, 356  
 exact sequence of modules, 55  
 exchange property of the multiplicity symbol, 306  
 extension formula for lengths, 168  
 extension formula for multiplicities, 333  
 extension of a module in a module of fractions, 136

## 442

- extension of an ideal, 86
- extension ring, 85
- external direct sum, 33
- factor complex, 357
- factor filtration, 383
- factor grading, 116
- factor module, 14
- field, 39
- filtered ring, 400
- filtration on a module, 379
- filtration topology, 380
- finitely generated module, 10
- formal power series, 28
- form ring of an ideal, 232
- fraction functor, 142
- free module, 59
- free module on a set of symbols, 61
- full matrix ring, 48
- full ring of fractions, 140
- functor, 141
- fundamental class of ideals, 256
- grade of an ideal, 241
- grade of an ideal on a module, 241
- graded radical of a graded ring, 233
- graded ring, 113
- graded ring associated with a multiplicatively filtered ring, 408
- grading monoid, 112
- Hausdorff space, 378
- Hilbert coefficients, 323
- Hilbert domain, 275
- Hilbert function, 317
- Hilbert module, 318
- Hilbert ring, 274
- Hilbert's Basis Theorem, 30
- homeomorphic spaces, 379
- homeomorphism, 379
- homogeneous element of a graded module, 114
- homogenous element of a graded ring, 113
- homogeneous ideal, 115
- homogeneous primary decomposition, 126
- homogeneous submodule, 115
- homology complex, 370
- homology modules of a complex, 352
- homomorphism of one module into another, 4
- homomorphism of one ring into another, 40
- $I$ -adic filtration on a module, 414
- $I$ -adic filtration on a ring, 414

## INDEX

- $I$ -adic functor, 423
- ideal of definition (for semi-local rings), 424
- identity element of a ring, 1
- identity mapping, 6
- image of a homomorphism, 10
- image of a set under a mapping, 10
- inclusion mapping, 6
- index set, 8
- induced filtration, 383
- induced grading on a homogeneous submodule, 116
- induced homomorphism, 15
- induced partial order, 71
- induced topology, 378
- inductive system, 71
- injection, 4
- integral closure of a ring in an extension ring, 89
- integral domain, 73
- integral element with respect to a ring, 86
- integral extension of a ring, 87
- integrally closed ring, 87
- internal direct sum, 31
- intersection of an empty family of subsets, 78
- Intersection Theorem, 206, 293
- inverse image of a set with respect to a mapping, 10
- irredundant decomposition, 104
- isolated component of a decomposable submodule, 107
- isolated set of prime ideals belonging to a decomposable submodule, 106
- isomorphic functors, 149
- isomorphic modules, 6
- isomorphism of complexes, 352
- isomorphism of one module on to another, 6
- isomorphism of one ring on to another, 40
- Jacobson radical, 207, 293
- Jordan–Hölder–Schreier Theorem for modules, 18
- kernel of a homomorphism, 5
- Koszul complex, 358
- Kronecker symbol, 61
- Lasker decomposition, 104
- leading coefficient of a polynomial, 266
- leading term of a polynomial, 266
- Lech's limit formula, 314
- left ideal, 26
- left module with respect to a ring, 2
- length of a module, 20
- lexicographical order, 128

- limit of a sequence, 385
- linear mapping of one module into another, 4
- localization at a prime ideal, 162
- localization principle for lengths, 166
- localization principle for multiplicities, 333
- local ring, 222
- Macaulay–Cohen ring, 257
- mapping of one pair of modules into another, 15
- maximal condition for submodules, 21
- maximal element of a partially ordered set, 71
- maximal ideal, 73
- maximal left ideal, 26
- maximal member of a family of submodules, 21
- maximal primary submodule, 202
- minimal condition for submodules, 23
- minimal prime ideal belonging to a decomposable submodule, 106
- minimal prime ideal of an ideal, 83
- minimal prime ideal of a ring, 84
- module with respect to a ring, 2
- monic polynomial, 87
- monomorphism, 4
- multiplication mapping, 155
- multiplicative filtration on a ring, 408
- multiplicatively closed subset of a ring, 75
- multiplicity symbol, 299
- multiplicity system, 295
- natural completion of a semi-local ring, 431
- natural mapping of a module on to a factor module, 14
- natural transformation of one functor into another, 149
- neighbourhood of a point, 378
- nilpotent element, 85
- Noetherian module, 181, 289
- Noetherian ring, 181
- normal decomposition, 104
- null homomorphism, 10
- null module, 8
- null ring, 1
- open set, 377
- opposite of a ring, 48
- order of a Hilbert module, 348
- partial order, 70
- polynomial in a single indeterminate, 29
- polynomial in several indeterminates, 178
- P*-primary ideal, 97
- P*-primary submodule, 98
- primary Artin ring, 190
- primary composition series, 202
- primary ideal, 97
- primary submodule, 97
- prime ideal, 73
- prime ideal belonging to a decomposable submodule, 106
- prime ideal associated with a primary ideal, 97
- prime ideal associated with a primary submodule, 98
- Principal Ideal Theorem, 217
- product of two homomorphisms, 4
- product of two ideals, 79
- product topology, 378
- projection of a module on to a direct summand, 32
- projective limit, 399
- proper submodule, 7
- pseudo-prime ideal, 350
- quasi-local ring, 163
- quaternions, 68
- quotient field of an integral domain, 140
- radical of an ideal, 81
- radical of a semi-local ring, 248
- rank of a prime ideal, 214
- rank of a proper ideal, 214
- refinement of a chain of modules, 18
- regular local ring, 350
- residue classes with respect to a submodule, 12
- residue field of a quasi-local ring, 223
- residue module, 14
- residue ring with respect to a (two-sided) ideal, 64
- right ideal, 26
- right module with respect to a ring, 2
- ring, 1
- ring of endomorphisms of a module, 39
- ring of fractions, 134
- R*-sequence, 235
- Samuel's limit formula, 329
- semi-local ring, 222
- semi-regular ring, 257
- semi-simple ring, 49
- separated space, 378
- simple module, 19
- simple ring, 49
- singly generated module, 10
- span of a module, 249

Cambridge University Press

978-0-521-09807-6 - Lessons on Rings, Modules and Multiplicities

D. G. Northcott

Index

[More information](#)**444**

strict homomorphism of filtered modules, 383  
 strongly compatible filtration, 408  
 subcomplex, 357  
 submodule, 7  
 submodule generated by a set, 9  
 subring, 85  
 subring generated by a set of elements, 87  
 subspace, 378  
 sum of a family of submodules, 9  
 supplement of a direct summand, 142  
 surjection, 5  
 system of generators of a module, 9  
 system of parameters, 226

torsionless grading monoid, 118  
 totally ordered set, 71  
 total order compatible with a monoid structure, 119

**INDEX**

trivial grading on a ring, 113  
 two-sided ideal, 26

unit of a ring, 222  
 unmixed ideals, 257  
 upper bound of a subset of a partially ordered set, 71

vector space, 167

Wedderburn's Theorem on simple rings, 50  
 well ordered set, 128  
 Wright's inequality, 296

zero-divisor, 74  
 zero-divisor on a module, 235  
 zero module, 8  
 Zeros Theorem, 285  
 Zorn's Lemma, 71