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D. G. Northcott
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LESSONS ON RINGS, MODULES AND MULTIPLICITIES

Lessons on rings, modules and multiplicities

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TO ROSE

who helped me most

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PREFACE

This book has grown out of lectures and seminars held at the University of Sheffield in recent years. Its purpose is to give a virtually self-contained introduction to certain parts of Modern Algebra and to provide a bridge between undergraduate and postgraduate study.

The title, *Lessons on rings, modules and multiplicities*, was chosen partly because a certain emphasis has been placed on instruction. I have long been interested in problems involving the introduction of young mathematicians to relatively advanced topics and, in this book, I have endeavoured to present the chosen material in a manner which will not only make it interesting but also easy to assimilate.

One fact of general interest has emerged which I did not foresee when I started. It was my intention to write about Commutative Algebra, but the contents of the first chapter are of such generality that it seemed wrong to exclude non-commutative rings at that particular stage. From then on the question continually arose as to the proper place at which to assume commutativity, and, indeed, the precise form the assumption should take. The outcome has been that this book, particularly in its later stages, is often concerned with Quasi-commutative Algebra. By this I mean that non-commutative rings are allowed but the emphasis is on the behaviour of central elements. In fact much that one normally regards as belonging to Commutative Algebra can be accommodated comfortably within this framework. For example, this is true of considerable areas of Multiplicity Theory and the theory of Hilbert Functions. It is also true of the theory of I-adic Completions and, to some extent, the theory of Primary Decompositions, though the latter fact gets only a passing mention in the exercises. Other instances where this observation is valid will doubtless occur to the reader as he proceeds.

It is with pleasure that I take this opportunity to acknowledge many sources of help and information. Since the subject matter of the book is strongly slanted in the direction of Commutative Algebra it was inevitable that the writings of N. Bourbaki, M. Nagata, P. Samuel and O. Zariski should have a persistent influence. Those who are familiar with the literature will also recognize that the chapter dealing with the Koszul Complex owes much to the classic paper on

Codimension and Multiplicity by M. Auslander and D. A. Buchsbaum. In a similar manner, the chapter describing the properties of Hilbert Rings is based on papers by O. Goldman and W. Krull.

This book has also profited from the research investigations of recent postgraduate students at Sheffield University. In particular, some ideas involving Multiplicity Theory and Hilbert Functions, made use of here, first appeared in the doctoral theses of K. Blackburn, D. J. Wright and W. R. Johnstone though they are now more widely available in standard mathematical journals.

An author is very fortunate if he has someone who is willing to read his manuscript with care and make detailed comments. D. W. Sharpe has performed this labour for me with a thoroughness which is familiar to those who know him well. His observations and constructive criticisms ranged from matters of punctuation and assistance with proof-correcting to comments on the organization of whole chapters. A number of sections have been rewritten to incorporate improvements which he has suggested. In the later stages, P. Vámos also helped me in a similar way and the final version has gained by being modified to take account of his observations.

Finally my thanks go to my secretary, Mrs E. Benson, who typed the manuscript and remained cheerful when I changed my mind and asked to have considerable proportions done again. Without her help this book would have taken very much longer to complete.

D. G. NORTHCOTT

Sheffield
March 1968

SOME NOTES FOR THE READER

There are certain matters which will be quite clear if the text is read consecutively from the beginning, but which require comment if you are primarily interested in particular sections and wish to study them in isolation. For example, it is necessary to know that the term *ring* is always understood to include the existence of an identity element, and the definition of a ring-homomorphism requires that identity element be mapped into identity element. A further point is that a homomorphism $R \to R'$, where R and R' are rings, is called an *epimorphism* only in the case where the mapping is surjective, that is to say when each element of R' is the image of at least one element of R.

Next there are some extensive sections in which only commutative rings are considered and in these the adjective *commutative* is usually suppressed in order to avoid tedious repetition. To discover whether the results of a particular section are established only in the case of commutative rings, it is sufficient to refer to the general remarks at the beginning of the chapter in which the results occur. These remarks contain, among other information, identification of all sections which are subject to this restriction.†

It will be found that the main text provides rather full explanations and for this reason, does not provide opportunities for you to devise your own arguments. To remedy this situation some exercises have been included at the end of each chapter. These are designed to let you test your grasp of basic concepts as well as to add to the information provided by the rest of the book. Certain of the more useful results contained among the exercises are employed at a later stage; but wherever this is the case the result in question is always established in the course of the discussion.

Cross-references are made in the following manner. If there is a reference to (say) Theorem 5 and no chapter or section is specified, then the result quoted is to be found in the chapter where the reference occurs. Where in one chapter it is necessary to recall a result established in some other chapter, the appropriate section is always given. To illustrate this, suppose that you have been referred to Proposition 18, Cor. 1 of section (3.9). Then the result in question is

† Similar use is made of the remarks which precede each set of exercises.

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the first corollary of the eighteenth proposition in Chapter 3. To narrow the search, the extra information provided says that it occurs in the ninth main subdivision of that chapter.

The final point concerns notation used in connection with sets. If X and Y are sets and X is contained in Y, then the symbol $X \subseteq Y$ is used to indicate this fact. The advantage gained is that when X is strictly contained in Y, that is to say when $X \subseteq Y$ and $X \neq Y$, it is possible to indicate this by writing $X \subset Y$. Although this conflicts with common practice, it will be found convenient and not a source of confusion.