

CONTENTS

<i>Preface</i>	<i>page v</i>
1 Theory of sets	
1.1 Sets	1
1.2 Mappings	3
1.3 Cardinal numbers	5
1.4 Operations on subsets	9
1.5 Classes of subsets	14
1.6 Axiom of choice	19
2 Point set topology	
2.1 Metric space	23
2.2 Completeness and compactness	29
2.3 Functions	35
2.4 Cartesian products	38
2.5 Further types of subset	41
2.6 Normed linear space	44
2.7 Cantor set	49
3 Set functions	
3.1 Types of set function	51
3.2 Hahn–Jordan decompositions	61
3.3 Additive set functions on a ring	65
3.4 Length, area and volume of elementary figures	69
4 Construction and properties of measures	
4.1 Extension theorem; Lebesgue measure	74
4.2 Complete measures	81
4.3 Approximation theorems	84
4.4* Geometrical properties of Lebesgue measure	88
4.5 Lebesgue–Stieltjes measure	95
5 Definitions and properties of the integral	
5.1 What is an integral?	100
5.2 Simple functions; measurable functions	101
5.3 Definition of the integral	110
5.4 Properties of the integral	115
5.5 Lebesgue integral; Lebesgue–Stieltjes integral	124
5.6* Conditions for integrability	127

iv	CONTENTS	
6	Related spaces and measures	
6.1	Classes of subsets in a product space	<i>page</i> 134
6.2	Product measures	138
6.3	Fubini's theorem	143
6.4	Radon–Nikodym theorem	148
6.5	Mappings of measure spaces	153
6.6*	Measure in function space	157
6.7	Applications	162
7	The space of measurable functions	
7.1	Point-wise convergence	166
7.2	Convergence in measure	171
7.3	Convergence in p th mean	174
7.4	Inequalities	183
7.5*	Measure preserving transformations from a space to itself	187
8	Linear functionals	
8.1	Dependence of \mathcal{L}_2 on the underlying $(\Omega, \mathcal{F}, \mu)$	194
8.2	Orthogonal systems of functions	199
8.3	Riesz–Fischer theorem	202
8.4*	Space of linear functionals	209
8.5*	The space conjugate to \mathcal{L}_p	215
8.6*	Mean ergodic theorem	219
9	Structure of measures in special spaces	
9.1	Differentiating a monotone function	224
9.2	Differentiating the indefinite integral	230
9.3	Point-wise differentiation of measures	236
9.4*	The Daniell integral	241
9.5*	Representation of linear functionals	250
9.6*	Haar measure	254
	<i>Index of notation</i>	261
	<i>General Index</i>	263