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† Note that the symbol c has two distinct uses, which should not be confused.

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[More information](#)**262** χ_E , 12 \mathbb{N}_0 , 6 $\in, \notin, \subset, \Rightarrow$, 1 \emptyset , 2 Π , 3 \exists , 4 \sim , 6 \cup, \cap, Δ , 9**INDEX OF NOTATION** \cup, \cap , 10 $\| \cdot \|$, 45 \int , see chapter 5 \ll , 148 \forall, \wedge , 241The symbol **■** is used to signal
the end of a proof.

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