AN INTRODUCTION TO HOMOLOGICAL ALGEBRA

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BY

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CONTENTS

PREFACE

The past ten years or so have seen the emergence of a new mathematical subject which now bears the name Homological Algebra. To begin with, it was the concern of a few enthusiasts in certain specialized fields but, since the publication of Cartan and Eilenberg's now famous book,† its importance for several of the main branches of pure mathematics has been generally recognized.

The young mathematician, about to start on research, will be anxious to learn about homological ideas and methods, and one of the aims of this book is to help him to get started. In trying to cater for his needs, I have imagined such a reader as being familiar with the notions of group, ring and field but still relatively inexperienced in modern algebra. For him, the account given here is self-contained save in a small number of particulars which are mentioned below, and which need not discourage him.

An introduction to homological algebra must, of necessity, be an introduction to the book of Cartan and Eilenberg, for the student who wishes to go further will need to read their work; but much of great interest and value has been achieved even more recently, and some of this later work has been given a place in the following pages. The list of contents gives a fairly detailed picture of the main topics treated, but a few additional comments may be a help.

Chapters 1–6 develop, in a leisurely manner, the results that are needed to establish and illustrate the theory of derived functors, after which follows an account of torsion and extension functors. These are the most important ones which are obtainable by the process of derivation and, in a sense, the remainder of the book is concerned with their applications. Such an application is the theory of global dimension given at the end of Chapter 7, and here are included some important results of M. Auslander on Noetherian rings that have previously been available only in the original research paper.

Chapter 9 deals with the structure of commutative Noetherian rings

† H. Cartan and S. Eilenberg, *Homological Algebra* (Princeton University Press, 1956).

of finite global dimension and represents one of the most satisfying achievements of homological methods. This, too, appears in a text-book for the first time. Here, it must be admitted, the account is not completely self-contained, but considerable care has been taken in explaining the results of Ideal Theory which are needed to supplement the purely homological arguments. This is the most ambitious chapter, and the author hopes that it will help to stimulate interest in commutative algebra. The treatment given here was found successful in a course of lectures in which the audience had no specialized knowledge of classical Ideal Theory.

Chapter 10 is an introduction to the homology and cohomology theories of monoids and groups. This, by itself, has a considerable literature and was one of the earliest branches of our subject to be developed. The chapter can be read, if desired, before Chapter 9 and does not require any specialized knowledge of Group Theory.† In deciding how far to go with this topic, I had in mind the student who might wish to acquire some general background before proceeding to the applications in some specialized field such as Class Field Theory.

Nearly all the topics covered in the following pages were included in a course of lectures given at Sheffield University. When lecturing, it is possible to digress at some length in order to explain the general plan of development and the connexions with other branches of mathematics. Also one likes to mention important results connected with what one is discussing even if there is no time for a full treatment. Some of this supplementary material, which I hope will add to the enjoyment and interest of the main text, will be found in the Notes which follow Chapter 10.

The final chapter has been much improved as the result of suggestions of J. Tate with whom I had an opportunity of discussing it. At Sheffield, I have been aided, at all stages, by my colleague H. K. Farahat. Of particular value has been his willingness to discuss points of detail and to make helpful criticisms. This work owes a great deal to his continued interest. I am also indebted to Sir William Hodge, who, when I first had the idea of writing an introduction to homological methods, encouraged me to go ahead.

† There is actually one reference to a result proved in Chapter 9, but there is no difficulty in taking this out of context.

Writing a book takes up much time and energy, and this one could never have been completed without the generous help of J. J. Kiely who typed the first draft from notes taken at lectures. I am also greatly indebted to my secretary, Mrs M. Ludbrook, for the great care and patience with which she cut innumerable exquisite stencils. To both of these I wish to express my thanks. Their strenuous efforts made it unthinkable not to finish a work to which they had contributed so much.

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NOTE ON CROSS-REFERENCES

If the reader is referred to a result, say to Theorem 7, and no chapter or section is specified, then he is to understand that the reference is to Theorem 7 of the chapter in which he is reading. When a reference is made in one chapter to a result which occurs in another, the section in which it will be found is also given. Thus Lemma 4 of section (5.3) means the fourth lemma in Chapter 5, and this will be found in the third subdivision of the chapter.