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PREFACE

It is well known that fundamental physical systems should always be Hamiltonian. In the 19th century Hamiltonian formalism was considered only as a secondary result of Lagrangian formalism, and it was written in the so-called "canonical" variables. Felix Klein thought that Hamiltonian formalism was beautiful, but useless for practical purposes. However, Henri Poincaré did use it in his famous work "Les méthodes nouvelles de la mécanique celeste". Differential forms appeared in the papers of Élie Cartan as a byproduct of the ideas of Poincaré. Much later (in the sixties) symplectic geometry was introduced as a very useful geometrically invariant way to understand some aspects of Hamiltonian formalism.

In the last twenty years, as a part of the famous achievements of soliton theory, a field-theoretical Poisson geometry has been developed. This geometry is extremely useful for the study of integrable systems described in terms of ordinary differential equations, partial differential equations and quantum field theories: Poisson geometry is much more useful in investigating these problems than symplectic geometry itself. I hope to convince the reader of this basic fact even in the comparatively simple cases examined in the first three Lectures, in which some beautiful finite-dimensional Hamiltonian systems are considered.

The main tool will be the study of the so-called hydrodynamic type systems, described in terms of first-order quasi-linear partial differential equations. About 150 years ago Riemann started studying them in connection with classical gas dynamics (dynamics of a compressible liquid). He observed geometrically invariant tensor structures in these systems. However, the fundamental question – When are these systems Hamiltonian? – has never been seriously investigated before the modern soliton theory.

The need of further developments in the study of these systems, which gave rise to the contemporary theory, appeared in the analysis of the well-known non-linear semi-classical approximation of quantum field theory (Whitham method) for the Korteweg-de Vries equation and other famous integrable systems. The Hamiltonian formalism and the criteria for integrability for these systems were found about 10 years ago, leading to beautiful constructions in differential geometry and algebra. Lectures 4 and 5 are devoted to the exposition of these ideas. This exposition is incomplete and proofs are often omitted (but references containing full proofs are given).

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