A handbook of terms used in algebra and analysis

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Preface

The enormous increase in mathematical activity and knowledge during the past half century has not been achieved without a corresponding increase in the number of terms which mathematicians use. Not only have names had to be given to new concepts and objects, for example categories and functors, but there has also been a need to attach new and/or more precise meanings to certain terms such as 'function' which have been used, in some sense or other, for centuries. Again, deeper insight into mathematical structure has often yielded an alternative way of approaching such well-established ideas as that of a derived function. This creation of new definitions and rewriting of old ones has not made life any the easier for the reader of mathematics. Certainly, an explanation of any of the terms can be found somewhere, but finding the correct source can be time consuming. The attractions of a reference book of definitions are, therefore, obvious.

Alas, the difficulties of compiling such a work are no less obvious! As soon as such a book were to appear it would be months out of date, for each issue of every mathematical journal can be expected to introduce at least one new term to the vocabulary of mathematics and frequently a new symbol – or an alternative usage of an old one – to accompany it. Moreover, talents equal to those of the troops of Bourbaki would be required to produce a comprehensive and authoritative work.

The objectives of this handbook, then, must be somewhat circumscribed. It is, for example, intended to meet the needs of the undergraduate and the school teacher rather than the university lecturer or the research student. It is concerned only with algebra and analysis. Nevertheless, I hope that within these limits the handbook will prove of value and that by its arrangement into sections, its examples and its notes, it will give some indication of, and some feeling for, the mathematics that has given rise to the definitions listed. Some cynics have asserted that 'modern mathematics' is 'all definitions', and it cannot be denied that it does contain a great number of them. It must be stressed, therefore, that the terms defined here are only the 'words' of mathematics and that a mathematician is interested not only in

learning new 'words' but in 'creative writing' and in appreciating the subject's 'literary heritage'.

Presenting a definition out of context is not a simple matter. 'How do I get to Newcastle from here?' is a question which has many answers. Is the questioner travelling on foot or on horseback, by bicycle or by car, or, come to that, by boat? Does he want the fastest route or the least congested? Would he like to see the new motorway, or travel on minor roads past some fascinating historical monuments? These questions all have their mathematical analogues – as does the apocryphal Irishman's answer that 'If I wanted to go to Newcastle, I shouldn't start from here.' There is always the chance, too, that the questioner really wanted to go to Newcastle-under-Lyme and has been directed in error to Newcastle-upon-Tyne. In an attempt to meet some of the analogous mathematical problems, many terms have been defined in this handbook in alternative ways; for example, 'continuity' is discussed in three different sections, namely, Metric spaces, Topological spaces, and Real-valued functions of a real variable. Theorems too have often been stated at alternative levels of generality. I have also tried to draw attention to words and phrases which are used in different senses by different authors, for example 'ring' and 'set of natural numbers'.

It would, of course, be difficult to answer the traveller without using numbers, yet no one is likely to preface his reply by an account of how the number system is constructed. In a similar manner, one cannot define terms in mathematics without using some words or symbols taken from logic. For this reason, the first section of the handbook describes those words and symbols which we shall wish to borrow from logic, without, however, concerning itself with the foundations of that subject.

The use of symbols in mathematical literature is, if anything, even more bewildering than that of defined terms. Only the '+' sign in the context of the addition of real numbers springs to mind as a symbol used by all writers in an unambiguous manner. (Lest readers press the case of the companion '−' sign, I was informed by a Scottish Inspector of Schools that he visited one primary school where the teacher taught the class that '−' was to be translated as 'from' and that the children were, therefore, to write equations such as $2-8=6$. The teacher explained to the inspector that the children experienced less difficulty with this convention!) I have tried, therefore, when introducing symbols to list any alternatives which are widely used.

In particular, in the sections on differentiation I have presented definitions, results and examples in a variety of notations – both 'ancient and modern' – in the hope that the opportunities thus provided for comparing the different notations will more than compensate for any possible loss in clarity.

Whilst producing this book I have received help and encouragement from many sources. As far as content is concerned, I am indebted to a host of mathematicians ranged alphabetically from Abel to Zorn and chronologically from Pythagoras to Cohen! Several colleagues or former colleagues at Southampton have been kind enough to read sections of the book and to offer most valuable advice. In particular, I should like to acknowledge my especial indebtedness to Professor T. A. A. Broadbent and to Dr Keith Hirst who read the final manuscript and whose observations led to the removal of several errors and, I hope, to a more readable book. I am also most grateful to Jennifer, my wife, for – amongst many other things – her assistance in the compilation of the index to this book.

May, 1971 A. G. HOWSON