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W. M. Smart

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COMBINATION OF OBSERVATIONS

BY

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CONTENTS

Preface *page* xiii

Chapter 1: FREQUENCY DISTRIBUTIONS

1·01	Introduction	1
1·02	Frequency distributions	2
1·03	Illustration of the practical application of statistics	4
1·04	Characteristics of a frequency distribution	6
1·05	Calculation of the mean	8
1·06	Moments	10
1·07	Formulae for the moments in terms of the class interval c	13
1·08	Example of the calculation of moments	13
1·09	Skewness and kurtosis	15
1·10	Relative frequency	17
1·11	Continuous frequency distributions	17
1·12	Moments for continuous frequency distributions	19
1·13	The normal frequency distribution	21
1·14	Integrals associated with the normal function	22
1·15	Properties of the normal distribution function	26
1·16	Example of a normal distribution	29
1·17	Sheppard's corrections	32
1·18	Note on the evaluation of $\operatorname{erf} t$	33

Chapter 2: ERRORS OF OBSERVATION AND THE PRINCIPLE OF LEAST SQUARES

2·01	Introduction	35
2·02	Instrumental errors	35
2·03	Personal equation	37
2·04	Systematic error	38

vi	<i>Contents</i>	
2·05	Accidental error	<i>page</i> 39
2·06	Summary of errors in general	40
2·07	Combination of observations	40
2·08	Observational equations in one unknown	41
2·09	The postulate of the arithmetic mean	41
2·10	Residuals	43
2·11	The principle of least squares	43
2·12	Standard error	45
2·13	The formula $\mu^2 = \frac{[vw]}{n-1}$	47
2·14	Illustrative example	48
2·15	Observations in which a constant systematic error is present	53
2·16	The standard error of a linear function of two independent variables	54
2·17	Hypothesis of elementary errors	55
2·18	The normal law of errors	56
2·19	The modulus of precision	58
2·20	Test of the normal law of errors	59
 <i>Chapter 3: PROBABILITY AND THE NORMAL LAW</i>		
3·01	Probability	62
3·02	Compound probability	62
3·03	Gauss's derivation of the normal law of errors	63
3·04	The postulate of the arithmetic mean	65
3·05	Continuous probability	65
3·06	Herschel's derivation of the normal law	66
3·07	The law of errors for a linear function	69
3·08	The law of errors for a linear function (alternative proof)	71
 <i>Chapter 4: MEASURES OF PRECISION</i>		
4·01	Introduction	73
4·02	The normal law	74

<i>Contents</i>		vii
4·03	The formula $\mu = \frac{1}{h\sqrt{2}}$ for a continuous distribution	<i>page</i> 74
4·04	The formula $\mu = \frac{1}{h\sqrt{2}}$ for discrete observations	75
4·05	The formula for μ in terms of the squares of the residuals	76
4·06	The standard error of the arithmetic mean	77
4·07	The formula $\eta = \frac{1}{h\sqrt{\pi}}$	78
4·08	Peters's formulae for μ and μ_a in terms of the numerical values of the residuals	78
4·09	Probable error	80
4·10	Remarks on measures of precision	81
4·11	Summary of rules	82
4·12	Example 1: the position angle of a double star	83
4·13	Example 2: the mechanical equivalent of heat	84
4·14	The standard error and probable error of a linear function of independent variables	85
4·15	Illustrative example	87
4·16	Precision of the standard and probable errors	88
 <i>Chapter 5: MEASURES OF PRECISION FOR WEIGHTED OBSERVATIONS</i> 		
5·01	Weighting of observations	93
5·02	Weighting according to the number of observations in a set	93
5·03	Arbitrary weighting	95
5·04	Weighting according to standard errors or probable errors	96
5·05	The weighted mean is the most probable value of the unknown	98
5·06	Reduction of weighted equations to equations of unit weight	99
5·07	The formula $\mu_0^2 = \frac{[wv]}{n-1}$	100
5·08	The precision of the weighted mean	101
5·09	Summary of formulae	102
5·10	Examples	103

Chapter 6: EQUATIONS OF CONDITION IN SEVERAL UNKNOWNNS

6-01	Linear equations of condition	<i>page</i> 107
6-02	Equations of condition in functional form	107
6-03	Equations of different weights	109
6-04	Normal equations	110
6-05	Checks in forming normal equations	112
6-06	Example of deriving normal equations (Gauss)	112
6-07	Residuals	115
6-08	Gauss's example (evaluation of $[vv]$)	116
6-09	Formal solution of the normal equations	117
6-10	Properties of α , β and γ	119
6-11	Summary of formulae	120
6-12	The formula $\mu^2 = \frac{[vv]}{n-3}$	121
6-13	Precision of x_0 , y_0 and z_0	123
6-14	Gauss's example (precision of solutions)	124
6-15	Gauss's method of solving the normal equations and evaluating the weights of the unknowns	126
6-16	Gauss's example (solution by Gauss's method)	130
6-17	Checks for Gauss's method	132
6-18	Alternative method of calculating weights	134
6-19	Equations of condition in two unknowns	135
6-20	Example (two unknowns)	136
6-21	Equations of condition with more than three unknowns (Gauss's method)	139
6-22	Other methods of solution	142
6-23	Evaluation of the unknowns in equations of condition connected by rigorous equations	144
6-24	Illustrative examples	147
6-25	Precision of a function of several unknowns determined from n equations of condition	150
6-26	Precision of the coordinates of the solar apex and of the solar velocity derived from measures of radial velocity	152

<i>Contents</i>	ix
<i>Chapter 7: THEORETICAL FREQUENCY DISTRIBUTIONS</i>	
7·01 Calculated and theoretical moments	<i>page</i> 155
7·02 Representation of a frequency distribution by a polynomial	156
7·03 Representation of a frequency distribution by a trigonometrical series	157
7·04 Precision of the constants	159
7·05 The Gram-Charlier series	160
7·06 Transformation of the Gram-Charlier series	161
7·07 Hermite's polynomials	162
7·08 Expression of the constants a_n and A_n in terms of the principal moments	164
7·09 The characteristics of the Gram-Charlier series	166
7·10 Theoretical frequency distributions derived from an assumed differential equation	166
7·11 Standard form of the differential equation	167
7·12 The theoretical principal moments	168
7·13 Skewness and inflexion	170
7·14 The theoretical distributions	170
7·15 Curves with mode at origin	171
7·16 Curves when $b = 0$	175
7·17 Curve when the roots of the auxiliary quadratic are imaginary	176
7·18 Curve when the roots of the auxiliary quadratic are equal	177
7·19 Curves when the roots of the auxiliary quadratic are unequal	179
7·20 Example of a Pearson curve	185
<i>Chapter 8: CORRECTION OF STATISTICS</i>	
8·01 Possible rejection of an observation	189
8·02 Chauvenet's criterion	190
8·03 Sheppard's corrections to moments	191
8·04 The correction of an observed frequency distribution	195
8·05 Eddington's solution of the integral equation	197

x	<i>Contents</i>	
8·06	Solution in terms of differences	<i>page</i> 197
8·07	Illustration of the correction of a frequency distribution	198
8·08	'Improved' value of a measure	199
8·09	The integral equation	200
8·10	Fourier transforms	200
8·11	Solution of the integral equation	202
8·12	Application to normal functions	203
8·13	Application to the Gram-Charlier series	204
8·14	Correction of vectors	206
8·15	Correction of the mean of the observed magnitudes of vectors	209
8·16	Relative frequency of the magnitudes of vectors exceeding a given value	210
8·17	Example of the correction of the mean observed magnitude of vectors	211
8·18	'Improved' value of the magnitude of an observed vector	212
8·19	Correction of a function of vectors by the method of operators	215
 <i>Chapter 9: CORRELATION</i>		
9·01	Introduction	217
9·02	Covariance	218
9·03	Lines of regression	220
9·04	The correlation coefficient	222
9·05	Example on the calculation of the correlation coefficient	223
9·06	Contingency tables	224
9·07	Example on the use of a contingency table	225
9·08	Parameters for continuous bivariate distributions	228
9·09	Normal bivariate distributions	229
9·10	The constants of the normal bivariate distribution in terms of σ_x , σ_y and r	231
9·11	The probability ellipse	234
9·12	Curved lines of regression for discrete distributions	235

Cambridge University Press
 978-0-521-09609-6 - Combination of Observations
 W. M. Smart
 Frontmatter
[More information](#)

<i>Contents</i>	xi
9·13 Correlation ratio for a continuous bivariate distribution <i>page</i> 237	237
9·14 Contingency coefficient	238
9·15 The contingency function ψ for a normal bivariate distribution	240
9·16 Normal trivariate distributions	242
9·17 The constants of the normal trivariate distribution	243
<i>Appendix 1.</i> VALUES OF $\operatorname{erf} t = \frac{2}{\sqrt{\pi}} \int_0^t e^{-t^2} dt$	246
<i>Appendix 1(a).</i> VALUES OF $(1 - \operatorname{erf} t)$	248
<i>Appendix 2.</i> VALUES OF $G(q)$	249
<i>Appendix 3.</i> VALUES OF $e^{-k^2x^2}$	250
<i>Index</i>	251

Cambridge University Press

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Frontmatter

[More information](#)

PREFACE

The subject-matter of this volume may be summarised briefly as the study of frequency distributions encountered in many branches of experimental science, and the treatment of observations and measures in which the incidence of errors is recognised. Most topics in the book have formed the substance of lectures for students proceeding to an Honours Degree in the University of Glasgow, during the past two decades.

Chapter 1 deals with frequency distributions as viewed against the background of general statistical theory; the various techniques relating to moments are considered in some detail, together with the integrals, associated with the normal function, which form the basis for many of the subsequent investigations.

The next five chapters are concerned with the treatment of observations and measures subject to errors, the general foundation being the normal law of errors. It is not too much to affirm that an observational or experimental result is to be judged in the absence of systematic error, not by its apparent agreement or disagreement with some other result previously obtained, but on the ascertainable degree of precision of the actual observations or measures relating to the investigation concerned. It is then necessary that the rules for deriving the degree of precision of a particular result should be expressed in some standard form which has universal sanction. In much of the earlier literature one of the measures of precision is known by the cumbrous expression of 'root-mean-square error'; I have replaced this important quantity by the simpler expression of 'standard error' which brings it into line with its counter-part, 'standard deviation', in the theory of statistics. Proofs of the normal law of errors and associated theorems are given in these chapters, with due regard to the various hypotheses which it is necessary to introduce if a mathematical formulation is to be achieved.

In several examples worked out in the text, one being intentionally of a highly artificial character, my aim has been to introduce arithmetic simplicity so that attention might not be diverted, by elaborate computational details, from fundamental principles and practical methods of procedure; the complete investigation of Gauss's well-known example involving three unknowns (Chapter 6) is a case in point.

Chapter 7 deals with the representation of a frequency distribution in various ways according to the nature of the series of measures recorded in particular observations or experiments, the object being

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W. M. Smart

Frontmatter

[More information](#)

xiv

Preface

to derive a mathematical expression which best represents the practical results; in particular, there are detailed discussions of the representation of measures by a Gram-Charlier series and of distributions derivable from Pearson's differential equation, with an example of the latter taken from current physical literature.

Chapter 8 deals with the correction of frequency distributions, the subject-matter including Sheppard's corrections to moments, the application of Fourier transforms and the correction of vectors.

The final chapter introduces the subject of correlation, the use of contingency tables, and a discussion of normal bivariate and tri-variate distributions.

In the appendices are to be found tables of several functions including, in particular, the error function; as regards the latter I am indebted to the Royal Society of Edinburgh for permission to make use of the elaborate tables calculated by J. Burgess (*Trans. Roy. Soc. Edinb.* **39** (1898), 257–321).

It may be added that no elaborate equipment is necessary for the statistical conduct of the various types of investigation with which the book deals; *Barlow's Tables*, *Crelle's Rechentafeln* (multiplication tables) the usual collection of logarithmic tables, and possibly a calculating machine, suffice for the arithmetic computations.

In conclusion I am again much indebted to the officials and staff of the Cambridge University Press for their care and attention during the course of printing.

W. M. S.

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