CAMBRIDGE

Cambridge University Press
978-0-521-09594-5 - Macroscopic Theories of Superfluids
Edited by G. Grioli
Excerpt
More information

Opening Address

EDOARDO AMALDI

Dear Colleagues and Friends,

I'm very honoured and pleased to welcome you on behalf of the Accademia Nazionale dei Lincei in this old building and to express my gratitude to all the participants who have come from various universities and laboratories.

I would also like to express my thanks to the Organizing Committee and, in particular, to its coordinator, Professor Grioli.

In the short time allotted by the programme to the Opening of the meeting, I will try to give you some information about this Academy, since I expect that you don't know much about it.

The Academy of the Lincei was founded in 1603 by four young people: 3 Italians and one Dutch man, a fact that established a connection between Italy and the Netherlands from the beginning: the leader of this group was Prince Federico Cesi. Last year the Academy celebrated the fourth centenary of his birth and next Saturday a meeting will be held at the Farnesina, the building on the other side of the street. On that occasion a very interesting exhibition of documents will be open to the public related to the activities and lives of Federico Cesi and his colleagues, in particular Jan Eyck, this young Dutch man who, as I said before, was one of the founders of the Academy.

The Academy was founded because, as the chronicles say, these young people were enflamed by the teaching of Galileo and the desire to spread his methods. (I don't know whether it makes sense to say enflamed in English, but I will say it all the same in the hope you can understand). Thus they started this Academy and there are a lot of documents from that period showing what they were doing. It is really very interesting.

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-09594-5 - Macroscopic Theories of Superfluids
Edited by G. Grioli
Excerpt
More information

Unfortunately the Academy was immediately accused of being a place where these young people were brewing obscure plots, and so a couple of years later it was closed. The Dutch man was taken on horseback out of Italy to Ventimiglia and ordered not to enter Italy again.

Fortunately Prince Cesi was very wealthy and so was able to finance this man, who went through Europe visiting many cities and famous places, spreading the idea that the methods of Galileo had to be pursued by international exchange of information and opinions.

This story shows that the idea of international collaboration goes back at least to this time. Later Jan Eyck was allowed to return to Rome and in 1611 Galileo also became a member of the Academy and from then on he used to sign himself Galileo Galilei Linceus. For instance his paper on sunspots as well as the *Sidereus Nuncius* was published by the Academy of the Lincei and both of them are signed as I said before.

The Academy of the Lincei was closed two or three times during its long lifetime. The last time was during the Fascist period, not because it was seen as a base for obscure plots, but as a place that did not follow the line of the Fascist government closely enough. The Academy of the Lincei was reopened and re-established immediately after the war.

After this short historical diversion, I will pause for a few more minutes to speak of our present meeting. I am not a theoretical physicist and, as an experimentalist, I do not work on superconductivity.

During the last 15 years I have been searching for gravitational waves and, thus, I became a daily user of superconductivity in its various forms. As a result my point of view on the subjects that you will discuss in these days is that of an outsider who is aware of the importance of this field in general and who knows enough about it to admire the corresponding theories too.

The importance of these theories *per se* is quite clear, but many people claim that the real importance of a scientific field can be judged and appreciated only by examining its influence on other fields of research and knowledge. I do not know if this really is the most appropriate parameter, but what is certain is that the macroscopic theories of superfluids involve non-linear equations like those in the fields of plasma physics, thermodynamics, oceanography and general relativity and that, for example, the same mathematical techniques used for solving certain non-linear equations, discovered during the last ten

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-09594-5 - Macroscopic Theories of Superfluids
Edited by G. Grioli
Excerpt
More information

years, in the study of plasmas, have also found application in general relativity.

Another fundamental but obvious connection is the one between the classical theories of superconductivity, and the interpretation in the frame of the quantum field theory, which is the only one that can provide a complete and deep understanding of this surprising aspect of nature.

I have also heard from some of my friends that the classical theories even contribute to our comprehension of the phase transitions phenomena near the critical point.

I will stop here. Perhaps I have even spoken too much. I will pass the chair to Professor Grioli so that you can start your work. Thank you very much.

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-09594-5 - Macroscopic Theories of Superfluids
Edited by G. Grioli
Excerpt
More information

Elementary Excitations of Nonlinear Classical Fields and the Theory of Superfluid He^4

SETH J. PUTTERMAN AND PAUL H. ROBERTS

I. INTRODUCTION

As liquid He^4 is cooled through $T_\lambda = 2.17°K$ it undergoes a phase transition to a superfluid state in which a number of unusual macroscopic dynamical motions can occur [1,2,3,4,5]. These include thermally propagating waves (second sound), fountain effect, reaction stresses due to heat flow, frictionless flow and quantization of vorticity. Since the interaction between He^4 atoms is the simplest intermolecular force in nature a key physical question presents itself: how can this simplest possible system display such a rich spectrum of phenomena when compared with other, more complex, fluids such as water?

The most basic approach for describing He^4 would begin with Schroedinger's equation for about 10^{21} atoms per c.c., interacting according to the known two particle potential $u_0(r)$ shown in Fig 1. That such a microscopic or first principles approach would involve substantial difficulties even as regards small oscillations (sound waves) at 0°K is clear from Fig 1 where one notes that at the average interatomic spacing of 3.6Å the curvature of the interaction potential is negative. Since sound waves do indeed propagate in He^4 one is required to search for an explanation for a negative bulk modulus in terms of cooperative effects even at the microscopic level.

Quantum zero point motion is generally credited with causing the atoms to be so far apart, and therefore also for the fact that He^4 remains a fluid down to 0°K at the vapor pressure. The resulting large interatomic spacing and low density ($\rho \sim 0.14$ g/cm^2) could suggest a description in terms of a weakly interacting Bose gas. Such a picture is particularly tempting because the concomitant Bose condensation [1] provides an immediate interpretation of the two fluid model in terms of a quantum ground state condensate (superfluid component) and thermal excitations (normal component). The two fluid theory is by far the most successful hydrodynamic theory of He^4 but the difficulty with the Bose condensate view is that no one has yet experimentally

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-09594-5 - Macroscopic Theories of Superfluids
Edited by G. Grioli
Excerpt
More information

or theoretically demonstrated that He^4 at $T = 0°K$ has a non-zero Bose condensate or off diagonal long range order [6] in some reduced density matrix.

As these first principle approaches are plagued with calculational difficulties and since cooperative effects are essential even at the microscopic level one is motivated to focus attention on the continuum mechanical equations under the hypothesis that in a Bose fluid continuum mechanics provides the exact description at zero degrees [7]. In this direction, we will discuss two types of continuum equations; (i) the Euler equations and (ii) the Ginzberg-Pitaevskii (G.P.) [8] equation and its generalizations.

The first problem is to elucidate the nature of the excitations of these equations. Next the finite temperature state of the fluid will be viewed also as a solution to these same equations but one which contains many of these excitations. This procedure can be carried out in a classical (non-operator) fashion and we shall find that the two fluid model of He^4 is a consequence of the exact one fluid equation (i.e. G.P. or Euler). The additional velocity field characteristic of the two fluid theory is due to the existence of an approximately conserved property of the excitations; their adiabatic invariant of motion.

According to this approach second sound is a classical property of a system of waves interacting according to nonlinear fluid dynamics. We will apply this interpretation to derive the reaction stress due to heat flow and to predict the existence of propagating energy modes in an acoustically turbulent medium.

II. CONTINUUM EQUATIONS AND ELEMENTARY EXCITATIONS

For an Euler fluid at 0°K four macroscopic variables are a complete set. They can be taken as the mass density $\rho(\mathbf{r}, t)$ and the local velocity field $\mathbf{v}(\mathbf{r}, t)$. The equations of motion then follow from conservation of mass and Newton's law for a continuum:

$$\partial\rho/\partial t + \nabla \cdot \rho\mathbf{v} = 0 \tag{1}$$

$$\partial\mathbf{v}/\partial t + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla\mu_0(\rho) \tag{2}$$

where

$$d\mu_0(\rho) = (1/\rho)dP_0(\rho) \tag{3}$$

and $P_0(\rho)$ is the pressure and $\mu_0(\rho)$ is the chemical potential. The simplest excitation for these equations corresponds to a small amplitude disturbance $\delta\rho$, relative to the background density ρ_0 so that

$$\rho = \rho_0 + \delta\rho; \quad \delta\rho/\rho_0 \ll 1 \tag{4}$$

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-09594-5 - Macroscopic Theories of Superfluids
Edited by G. Grioli
Excerpt
More information

From (1, 2) one finds that in the linear approximation $\delta\rho$ can be expanded in terms of traveling waves of the form

$$\delta\rho = \rho' \exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t) \tag{5}$$

where

$$\omega/k = c_0(\rho) = (dP_0/d\rho)^{1/2} \tag{6}$$

A key aspect of a quantum fluid is the quantization of circulation [9]

$$\Gamma \equiv \oint \mathbf{v}\cdot d\ell = nh/m \tag{7}$$

where h is Planck's constant, n an integer, m the mass of a helium atom and the integral is taken round any closed contour moving with the fluid. While this condition is consistent with the Euler equations they do not demand it. It is sensible to formulate continuum mechanics in a fashion where (7) is unified with (1,2). This is achieved for example by the G.P. equation

$$i\hbar\partial\psi/\partial t = -(\hbar^2/2m)\nabla^2\psi + W_0|\psi^2|\psi \tag{8}$$

where the macroscopic wave function is

$$\psi = \rho^{1/2}e^{i\theta} \tag{9}$$

The imaginary part of (8) yields (1) provided we identify

$$\mathbf{v} = (\hbar/m)\nabla\theta \tag{10}$$

and the real part yields (2) with

$$\mu_0 = -\frac{\hbar^2}{2m^2}\frac{\nabla^2\rho^{1/2}}{\rho^{1/2}} + \frac{W_0}{m}\rho \tag{11}$$

For this equation the small amplitude dispersion law is

$$\omega^2 = {c_0}^2k^2 + (\hbar^2/4m^2)k^4 \tag{12}$$

where now

$$c_0^2 = W_0\rho_0/m \tag{13}$$

The G.P. equation thus yields an excitation spectrum of small oscillations that is similar to the Bogoliubov theory of the weakly interacting Bose gas [10].

Liquid He^4 must be different from this model because of the roton minimum (see Fig. 2) in the dispersion law, and a sound velocity which depends upon ρ

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-09594-5 - Macroscopic Theories of Superfluids
Edited by G. Grioli
Excerpt
More information

in a more complicated fashion than shown in (13). This leads us to emphasize a generalization of (8) to include dispersion and other more general deviations from Hook's law:

$$i\hbar\partial\psi/\partial t = -(\hbar^2/2m)\nabla^2\psi + \bar{\mu}(\rho)\psi + m^{-1}\int |\psi^2(r')|V(r-r')\psi(r)d\mathbf{r}' \quad (14)$$

for which the small amplitude dispersion law is [11]

$$\omega^2 = (\hbar^2/4m^2)k^4 + {c_0}^2k^2 + (2\pi)^3(\rho_0/m^2)V(k)k^2 \quad (15)$$

where

$$c_0^2 = \rho_0(\partial\bar{\mu}/\partial\rho_0)/m \quad (16)$$

and

$$V(k) = (1/2\pi)^3\int V(r)e^{-i\mathbf{k}\cdot\mathbf{r}}d\mathbf{r} \quad (17)$$

Single-valuedness of ψ in (8) or (14) leads to the quantization condition (7). For reasonable potentials (such as $V(r) = V_0\delta(r-a)$ for suitably chosen "a") the dispersion law (15) has a structure suggestive of the photon-roton dispersion law [12].

The Laplacian in the G.P. equations (8) or (14) leads not only to the conservation of mass (1) but also introduces an intrinsic dispersion not contained in the Euler equations (1,2). Consequently the G.P. equation and its generalization have elementary excitations in addition to the small oscillations. The energy ε versus momentum p for these excitations is shown in Figure 3 by the solid lines [13]. The dashed line follows the dispersion law implied by (12). At high p where the solid line follows $\varepsilon = O(p^{1/2})$ the system has a quantized vortex ring excitation [14]. As the ring shrinks so that ε, p diminish the momentum and energy pass smoothly through a value p_c, ε_c where the circulation abruptly disappears [13]. Between p_m and p_c on the lower branch one has a three dimensional solitary wave which like all the excitations travels with a velocity $\mathbf{v} = d\varepsilon/d\mathbf{p}$. This localized disturbance has no circulation or topological constraint and simulations of linear stability suggest that the lower branch is stable whereas the upper branch is unstable [15].

The upper branch has no circulation and corresponds to a 3 dimensional solitary wave that increases in size as p increases. At large p it becomes a disc-shaped rarefaction of extended spatial dimensions moving at the $k \to 0$ sound velocity.

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-09594-5 - Macroscopic Theories of Superfluids
Edited by G. Grioli
Excerpt
More information

According to the Kelvin circulation theorem the rate of change (D/Dt) of the circulation Γ as one follows the fluid particles making up a closed path integration vanishes or

$$D\Gamma/DT = 0.$$

In view of this theorem the instability of the upper branch and the excitation properties near p_c are at first a little surprising. One might wonder how an excitation without circulation (upper branch) can spontaneously decay into an excitation with circulation (lower branch). The clue to the resolution of this paradox is that the Kelvin theorem applies only to a path constituted by fluid particles. At the vortex core the superfluid density $\rho_s = 0$, thus when a path crosses a vortex core there is an instant when the path is broken and Kelvin's theorem ceases to apply. At this moment the circulation can slip without violating the Kelvin theorem.

In Figure 4 is indicated a sequence of computer generated snapshots of $|\psi(s, z, t)|^2$ for a disturbance slightly perturbed from the upper branch. Scaled time and cylindrical coordinates (s, ϕ, z) are used, and $|\psi^2|$ is scaled so that the contour $|\psi^2| = 10$ corresponds to the undisturbed fluid density. A meridional section is shown and each figure should be thought of as defining a surface obtained by rotating the section about the "vertical" axis of symmetry bounding each figure on the left. Between $t = 12$ and $t = 18$ the contour $|\psi^2| = 2$ pinches off from the axis of symmetry $(s = 0)$ and a circle of nodes (intersecting the section shown at a point inside the nested curves shown in the last figure) of ψ appears along with a non-zero local quantized circulation calculated along any contour surrounding this circle of nodes.

There are two unresolved issues regarding the continuum picture of elementary excitations discussed here. One is related to the fact that the lowest lying excitation spectrum for He^4 is continuous. The other is that in reality the vortex-like branch apparently attaches to the photon-roton (small oscillation) branch at finite k (A in figure 2). While the modifications in the above theory or analysis that will produce these properties is not known, we would like to suggest how a still more physical model might emerge.

As regards the continuity of the spectrum one can wonder if the inclusion of higher order nonlinear effects which mix waves and solitary excitations might make possible a smooth transition from the vortex-like branch to small oscillations. For the model (14) where nonlocal effects are included one would perhaps expect that the vortex branch will attach near the roton minimum where $d\omega/dk = \omega/k$. For the G.P. equation (8) the group and phase velocities

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-09594-5 - Macroscopic Theories of Superfluids
Edited by G. Grioli
Excerpt
More information

become equal only for $k \to 0$ but for the phonon-roton spectrum they also become equal at the Landau critical velocity. The interaction terms which lead to a continuous transition from vortex-like to small oscillation-like behavior could then be expected to repel the higher k, ω modes so as to introduce a gap as in Fig 2.

A successful resolution of these issues will yield insight into Onsager's view of the roton as "the ghost of a vanished vortex ring."

III. RELATION OF THE TWO FLUID THEORY TO THE ONE FLUID CONTINUUM EQUATIONS

At non-zero temperatures T, the fluid is permeated by a gas of elementary excitations. The continuum equations which describe the adiabatic transformation of the $T = 0$ state also describe the elementary excitations. Thus the one fluid continuum equations (e.g. 1,2) can be expected to contain the two fluid theory of He^4 [16]. The physical insight which helps to make this connection apparent is that the fluid properties $(\rho, \mathbf{v})$ of the excitations will vary on a length (time) scale short (fast) compared with the long wavelength macroscopic motions that are for instance determined by the boundaries. Our picture is that the exact solution to (1,2) or (8) or (14) will contain a long wavelength or background contribution in addition to short wavelength contributions determined by the excitations. The long wavelength piece will describe the superfluid motion and the short wavelength part will describe the normal fluid motion of the two fluid model. Since a one component theory (e.g. 1,2) describes all the dynamics one can expect that nonlinearities will couple the long and short wavelength motions. The key to deriving the two fluid model is to note that in the reversible limit this coupling will vanish in view of the fact that the excitations possess an adiabatic invariant of the motion. Thus at any instant in time the exact point to point description of the fluid is given by (1,2) or (8) or (14); and in the reversible limit this motion possesses a hidden conservation law so that on an averaged level an extra velocity field and the two fluid model emerge. This happens provided that T is not too large. At large enough T the background is fully depleted and one should arrive at a one fluid theory at the Navier-Stokes level.

For the purpose of developing this idea we will use the Euler equations. A more physical description of He^4 (above .8K) should follow from a model such as (14) which includes rotons and quantized vortex excitations. The approach below has been generalized to include effects due to dispersion [16] but the vortex-vortex and vortex-small oscillation interactions have not yet been incorporated into the overall framework of this one fluid approach.

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-09594-5 - Macroscopic Theories of Superfluids
Edited by G. Grioli
Excerpt
More information

Three key nonlinear effects determine the motion on various length scales:

A. The scattering of one sound wave by another to produce sum and difference frequencies and wave numbers.
B. The refraction of short wavelength sound waves by long wavelength background variations.
C. The reaction of the background so that total energy and momentum are conserved during the refraction.

As the distinction between normal and super components is based on the length scale of variation of $\rho, \mathbf{v}$ we expand $\rho(\mathbf{r}, t)$ in terms of a slowly varying background $\rho_0(\mathbf{r}, t)$ and high frequency waves with amplitudes ρ_n:

$$\rho = \rho_0 + \sum_{n=1}^{\infty} \int \rho_n(\mathbf{k}, \mathbf{r}, t) \exp[i\theta(\mathbf{k}, \mathbf{r}, t)] d^3k + c.c. \tag{18}$$

where c.c. denotes the complex conjugate and we have in mind an expansion in terms of a small parameter ε where $\rho_n \sim \varepsilon^n$ but $\nabla\rho_n$ and $\partial\rho_n/\partial t$ are of order ε^{n+2}; θ satisfies the differential form $d\theta = \mathbf{k} \cdot d\mathbf{r} - \omega dt$ where $\mathbf{k}, \omega$ are zeroth order in ε but their derivatives (e.g. $\partial\mathbf{k}/\partial t$) are second order. For simplicity the expansion of ρ_0 in powers of ε is not indicated. In parallel with (18) a similar expansion is applied to $\mathbf{v}$.

At second order in ε the contribution of a given wave to the acoustic energy density per volume d^3k of k space is

$$\varepsilon_k = (c_0^2/\rho_0)|\rho_1|^2 + \rho_0|v_1|^2 \tag{19}$$

Introducing the wave action $n(\mathbf{k}, \mathbf{r}, t) \equiv \varepsilon_k/\omega_0$ one finds from (1,2) at order ε^4

$$\left(\frac{\partial n}{\partial t}\right)_{k,r} + \left(\frac{\partial \omega}{\partial k_i}\right)\left(\frac{\partial n}{\partial r_i}\right)_{k,t} - \left(\frac{\partial \omega}{\partial r_i}\right)_k \left(\frac{\partial n}{\partial k_i}\right)_{r,t} = I(n) \tag{20}$$

where $\omega = \omega_0(k) + \mathbf{v_0} \cdot \mathbf{k}$ with $\omega_0 = c_0 k$. The left hand side of (20) describes the geometrical acoustic effect whereby high frequency waves are refracted by variations in the background (effect B). The right hand side of (20) describes the redistribution of sound wave energy due to nonlinear collisions between waves (effect A). The collision integral will be discussed later. However, it has the important property that at lower order (ε^4), $\mathbf{k}, \omega$ are additive, conserved Galilean invariants of an interaction of two sound waves so that to order ε^4

$$\int I(n)\mathbf{k} d^3\mathbf{k} = 0 \tag{21}$$

$$\int I(n)\omega d^3\mathbf{k} = 0 \tag{22}$$

© Cambridge University Press www.cambridge.org