CALCULUS: Basic concepts and applications

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Preface

Mathematics, with its origin in problems of land measurement and the keeping of accounts, has grown in complexity and power as the needs of society have required ever more sophisticated reasoning and techniques. Over a period of 2000 years mathematics developed, in some periods slowly, in others rapidly, until in the seventeenth century there was a dramatic advance – the invention of calculus – to match and facilitate equally dramatic achievements in science and, somewhat later, in technology.

Indeed, calculus proved essential for the handling of difficult problems in astronomy, physics, and engineering, as well as in other branches of mathematics itself, such as the determination of tangent lines to curves and the computation of volumes bounded by closed surfaces. In the eighteenth and nineteenth centuries, the demands of the physical sciences and technology stimulated rapid and far-reaching developments of the branch of mathematics called *analysis*, growing out of calculus; and, reciprocally, the mathematical developments contributed to the further growth of those sciences.

Comparable progress was not made in the applications of mathematics, including calculus, to the social and life sciences, largely because problems in these areas proved difficult to formulate in mathematical terms. With what precision – or even meaning – can one assign a number to the degree of a person's conviction on a controversial subject, such as the investment of more money to reduce the size of classes in public schools?

In the last 50 years, however, much of the mathematics that has proved so useful in the physical sciences has been applied successfully in the social and life sciences. The examples and problems presented in this book recognize this fact. We shall often find that the major difficulty is the proper *formulation* or *modeling* of the problem: Starting with a complicated and perhaps vaguely defined situation, how do we sharpen the definition of the problem, removing inessential features and adding data as required, so as to make it feasible to apply mathematical methods? Providing practice in the formulation of problems, as well as in their solution, is one of our goals in this book.

Thus far, we have mentioned mathematics solely in the context of its applications to various fields. But mathematics has also been created and studied for its own sake, as a system of thought with great appeal because of its logical and aesthetic qualities. In this sense, mathematics is an art, in addition to being a tool of the sciences. In its dual roles, mathematics holds a central position in our cultural heritage, and an appreciation of mathematics should contribute significantly to our intellectual development, in the same way that an appreciation of literature, music, and philosophy, for example, contribute to that development. We hope that study of this book will prove rewarding through an increased appreciation of the power and the beauty of mathematics.

How this book is organized, and how it can be used

The spirit of the development of calculus in this book is *intuitive*, with "real-world" problems and concrete examples to provide motivation and to clarify concepts. We have chosen data to minimize arithmetic and algebraic complexities while you are absorbing new ideas, and we use plausibility arguments rather than formal proofs to justify most of the conclusions. However, we have tried to make careful statements, so that you will never have to unlearn anything later.

Significance of ★

Chapter 0 provides a review of algebra, graphing, and related topics for those who need it. The core material of Chapters 1 through 7 is appropriate for a one-semester, three-hour course for an average class. Students with a strong background in mathematics may work the starred problems and study the starred sections, which go more deeply than the core material into some of the topics. (There are some proofs in starred sections.) Omission of the starred sections, however, does not interrupt the basic development of the subject.

Students with a more limited background can skip some sections as well:

Sections 3.4, 3.9, and 4.14 involving applications to economics;

Section 3.10 on approximate solution of equations; and

Sections 4.3, 4.8, 4.11, and 6.6 on extreme rates of change, related rates, and relative rates.

These can all be omitted without giving you trouble with subsequent material.

Chapter 8, Chapter 9, and the core of Chapter 10 can be studied, in any order, after Chapter 7 has been completed. Moreover, at the end of Chapter 7 we have made suggestions for independent projects that you may find of interest. On the assumption that you have developed some mathematical maturity in working through the first seven chapters, we have somewhat condensed the exposition in the later chapters. In those chapters, too, you will find considerable emphasis on numerical methods.

Significance of ○

In addition to problem sets at the end of sections, there are exercises embedded in the expository material itself. Be sure to do them, for they are designed to help you understand the material that follows. Other exercises are marked with a small open circle ○; you should do all these exercises and save your solutions, because results later in the book depend upon them.

At the end of each chapter, in addition to review problems, there is a set of questions constituting a "sample test," which should help you to check whether you have mastered the material. Answers to selected problems, and to virtually all the "sample test" questions, appear at the end of the book.

Significance of **C**

For problems marked with the letter **C** a modern calculator will be useful. This is not to say that you should avoid using a calculator on other problems, but rather that some of the numerical experimentation suggested by the **C** problems can be extremely time-consuming if done by "hand."

The use of calculators in the study of calculus has both advantages and drawbacks. Among the advantages are these: (1) many concepts can be well illustrated and simply explained through calculation, and (2) the practical applications of calculus often call for delicate numerical work. Among the drawbacks, we note that the intricacies of calculators can become a study in itself and a distraction from learning basic concepts.

This book seeks a middle ground. Its primary thrust is calculus, and the material is presented so that you can acquire all the essential content without the use of a calculator. At the same time, there is additional content – and opportunity for additional insights – for those who choose to use calculators.

Even the simplest calculator, with nothing more than a square-root key, will come in handy in saving you time and in providing you with significant insights. More useful is a calculator with the usual features of scientific-engineering models: logarithmic, exponential, and trigonometric functions, and floating point representation. Better yet is a programmable calculator, preferably with branching ability. Best of all is a computer – and a little knowledge of how to use it.

Some study hints

Your secondary-school course in geometry has given you some experience with the careful statement, the attention to detail, and the concern for logical argument that are typical of mathematics. You may or may not have approached algebra in a similar spirit. In calculus we *must* be precise in language, alert to the niceties of seemingly "minor" points, and prepared to follow a rigorous argument, because the material involves subtleties that must be appreciated if you are to learn to handle novel situations (as contrasted with merely solving routine problems) and to realize the aesthetic satisfactions and intellectual rewards that can come from a critical study of calculus and related topics.

In order to learn as much as possible from the text, you should form the habit of reading it slowly, while seated at a desk, with paper and pencil at hand. It is probably worthwhile to "skim" the assigned reading once, to obtain a general idea of the subject matter. Then study the material, reading each sentence slowly, and doing your best to fill in any details that we have left to you. Next, review the material, analyzing its relation to what you have previously learned and trying to put the main results into your own words. Finally, do problems and exercises.

Throughout your study you should maintain a critical attitude. Constantly ask questions: Why is it done this way? Could it not have been accomplished more easily as follows? Is this hypothesis really needed? Doesn't the following example contradict the statement in the text? Is this a significant or a trivial point, and what relation does it have to the entire structure that I'm trying to understand? And so forth. By cultivating an active involvement in the course you will greatly increase the satisfaction obtained from it.

Here is still another suggestion: When you finish working a problem, spend a few minutes thinking through what you have done. As soon as you have come out with a neat $r = 5$; $h = 10$, there is a temptation to think That's done; what's next in my assignment? But there is great value in reviewing a completed problem in terms like these:

What was asked for in this problem? Have I answered the question(s)? Does my answer sound right – does it make sense?

How do this problem and its answer compare with other problems I have solved and with situations I know apart from my math course?

Are there any general conclusions that I can draw from my work on this problem?

Now that I have solved the problem, do I see some easier way that it could have been done?

In short, what have I *learned* from this problem?

Such an analytical, reflective approach will pay big dividends in understanding and enjoyment.