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The numbers refer to pages. Reference is made to definitions of terms (on pages indicated here by *italic numerals*), to topics on which there are exercises, to important ideas that are discussed or mentioned, and to words whose mathematical uses are exemplified or explained. (Certain 'ordinary' English words have special meanings in mathematical contexts, and the correspondence between the words and their meanings is not one-to-one.) Some of the cross-references indicate significant linkages of ideas: the reader may find it instructive to consider the nature of these linkages, and to trace others.

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