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FOURIER TRANSFORMS

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BY

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PREFACE

The study of abstract harmonic analysis has been much in vogue for two decades or more. There is, however, no one place where the interested reader can learn the underlying classical theory in a form which immediately lends itself to an understanding of the abstract.

This tract is designed to provide a background in those classical theorems concerning Fourier transforms on the real line which have found fruitful generalization in abstract harmonic analysis.

Although a familiarity with Lebesgue and Riemann–Stieltjes† integration is required of the reader, we state in Chapter 1 all the theorems on integration used in the subsequent chapters. The reader should also have an acquaintance with the most elementary theory of functions of a complex variable.

In Chapter 2 we introduce the Fourier transform on L^1. After determining its fundamental properties we prove that an analytic function of a Fourier transform is locally a Fourier transform. This is used to establish Wiener's celebrated result on the closure of translates in L^1. To end the chapter, we give an 'algebraized' reformulation of some of the preceding results in terms of ideals in a commutative Banach algebra.

The next chapter is devoted to the Fourier transform on L^2. In particular, there is a proof of Plancherel's theorem.

In Chapter 4 we consider generalizations of the theorem of Wiener proved in Chapter 2. We take up the problem (equivalent to the famous spectral synthesis problem of Beurling) of whether or not the zeros of the Fourier transform of an L^1 function determine the span of the translates of the function.

Bochner's characterization of Fourier–Stieltjes transforms of non-decreasing bounded functions is the subject of the last chapter.

In addition, there is an appendix in which we briefly point out how many of the concepts and theorems can be carried over to an arbitrary locally compact abelian group.

† In the last chapter, only.

In order to keep the tract as elementary as possible, we have avoided all use of the methods of functional analysis. This has meant that we have occasionally had to satisfy ourselves with statements of (or references to) certain recent results which can be stated in classical language but which require functional analysis for any reasonable proof.

References in the text to the bibliography are given thus: [3], or [21; 25] (reference 3, or page 25 of reference 21).

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R. R. G.