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BY

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PREFACE

Although convexity is used in many different branches of mathematics there is no easily available account dealing with the subject in a manner which combines generality with simplicity. My aim in writing this tract has been to provide a short introduction to this field of knowledge for the use of those starting research or for those working on other topics who feel the need to use and understand convexity.

In a short tract, on a subject such as this, it is difficult to decide both the level of generality to aim at and the exact parts of the subject to omit. On the one hand, to accommodate the needs of economists and others, it is desirable to have available results that refer to n -dimensional real Euclidean space; on the other hand, more general spaces present such diverse characteristics that they cannot be conveniently dealt with in a tract of this size. For this reason the containing space is taken to be n -dimensional real Euclidean space except in the last two chapters. As to the subjects omitted there is nothing on the geometry of numbers, packing or covering problems, differential geometry on convex surfaces, integral geometry or the analogy with complex convexity.

The tract falls naturally into three parts. The first and third chapters contain the basic properties of individual convex sets and functions. The second chapter is an illustration of the way in which the comparatively simple properties obtained in the first chapter can be applied. In the fourth and fifth chapters convexity is investigated more fully, the properties of classes of convex sets are developed and the effects of certain operations on these classes are studied. The last two chapters contain examples of results and techniques in the solution of particular problems.

The notes at the end of the tract contain brief indications of the sources of the material in the tract and of suitable papers or books for further reading. Other bibliographies will be found in the books referred to there, in particular those by Bonnesen and Fenchel, by Hadwiger and by Fejes Tóth.

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PREFACE

My thanks are due to Dr F. Smithies, Fellow of St John's College, Cambridge for inviting me to write this tract and for reading the manuscript; and to Mr B. J. Birch, Fellow of Trinity College, Cambridge, for reading the proofs with a critical eye. Apart from correcting many minor mistakes both have made suggestions for improving the text that have been of great value.

H. G. E.

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