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978-0-521-09422-1 - Homology Theory: An Introduction to Algebraic Topology

P. J. Hilton and S. Wylie

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