

Cambridge University Press

978-0-521-09333-0 - An Engineering Approach to Linear Algebra

W. W. Sawyer

Frontmatter

[More information](#)

***An engineering
approach to
linear algebra***

Cambridge University Press

978-0-521-09333-0 - An Engineering Approach to Linear Algebra

W. W. Sawyer

Frontmatter

[More information](#)

An engineering approach to linear algebra

W. W. Sawyer

*Professor jointly to
the Department of Mathematics
and the College of Education
University of Toronto*

***Cambridge
at the University Press
1972***

Cambridge University Press
978-0-521-09333-0 - An Engineering Approach to Linear Algebra
W. W. Sawyer
Frontmatter
[More information](#)

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521084765

© Cambridge University Press 1972

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 1972
This digitally printed version 2008

A catalogue record for this publication is available from the British Library

Library of Congress Catalogue Card Number: 70-184143

ISBN 978-0-521-08476-5 hardback
ISBN 978-0-521-09333-0 paperback

Contents

Preface	<i>page</i> vii
1 Mathematics and engineers	1
2 Mappings	1
3 The nature of the generalization	4
4 Symbolic conditions for linearity	9
5 Graphical representation	12
6 Vectors in a plane	15
7 Bases	17
8 Calculations in a vector space	22
9 Change of axes	25
10 Specification of a linear mapping	28
11 Transformations	33
12 Choice of basis	38
13 Complex numbers	45
14 Calculations with complex numbers	51
15 Complex numbers and trigonometry	55
16 Trigonometry and exponentials	57
17 Complex numbers and convergence	63
18 Complex numbers: terminology	65
19 The logic of complex numbers	66
20 The algebra of transformations	68
21 Subtraction of transformations	73
22 Matrix notation	78
23 An application of matrix multiplication	89
24 An application of linearity	92
25 Procedure for finding invariant lines, eigenvectors and eigenvalues	99
26 Determinant and inverse	105
27 Properties of determinants	114
28 Matrices other than square; partitions	129
29 Subscript and summation notation	137
30 Row and column vectors	139
31 Affine and Euclidean geometry	144
32 Scalar products	147
33 Transpose; quadratic forms	156
34 Maximum and minimum principles	161
35 Formal laws of matrix algebra	169

Cambridge University Press

978-0-521-09333-0 - An Engineering Approach to Linear Algebra

W. W. Sawyer

Frontmatter

[More information](#)**vi CONTENTS**

36	Orthogonal transformations	176
37	Finding the simplest expressions for quadratic forms	186
38	Principal axes and eigenvectors	198
39	Lines, planes and subspaces; vector product	211
40	Systems of linear equations	235
41	Null space, column space, row space of a matrix	253
42	Illustrating the importance of orthogonal matrices	261
43	Linear programming	266
44	Linear programming, continued	280
	Answers	
	Index	

Preface

This book first took form as duplicated notes handed out to first year engineering students at the University of Toronto. The preparation of this material was undertaken because no published book seemed to meet the needs of first year engineers. Some books were mathematically overweight; in order to prove every statement made, long chains of propositions were included, which served only to exhaust and antagonize the students. Other books, emphasizing applications to engineering, involved mathematics beyond the student's comprehension. This was naturally so, for while linear algebra has a wealth of applications to engineering and science, many of these lie outside the range of the first year student, both in their mathematical and in their scientific content.

Thus problems of treatment arose both on the mathematical and the engineering side.

In some discussions of mathematics teaching, it seems to be assumed that only two courses are open: either the full rigorous proof of every statement from a set of axioms, or a cookbook approach in which the student memorizes some rigmarole. The end result of these two approaches is much the same; in neither case does the average student have any understanding of what he is doing. A third course is in fact possible, in which the student learns to picture or to imagine the things he is dealing with, and then, because his imagination is working, he is able to reason about these things. The objective of this book is to let the student achieve such insight and reasoning ability. I would strenuously oppose any suggestion that this book is less mathematical because it steers in the direction of this third course. Imagination is the source of all mathematics; a mathematician begins with an idea; the details of proofs come later. All the explanations, all the arguments, all the justifications brought forward in this book could, if one desired, be put in a rigorous form that would satisfy a professional mathematician, but it would not help the first year student if this were done.

If a mathematics course for engineers is to succeed, it must give the student the feeling that it will be of practical value in enabling him to master his profession. No mathematical concept has been included in this book without a careful examination of its value for applications, and the right of the student to demand evidence of such applicability has throughout been respected. The difficulty that most actual applications are very complicated has been met by discussing simplified situations, but indicating the existence of more realistic problems.

So far as I know, no other book uses the approach of the opening sections, which start with the real world and show how real situations lead to the basic concepts of linear algebra. The exercises at the end of §3 are intended to show the variety of situations in which these concepts are involved. Students should be warned that they will find these exercises hard. They should be prepared to bypass them to some extent,

viii PREFACE

to go ahead with the rest of the book, and to come back to these exercises from time to time.

Some books are written to provide problems and extra reading for students whose main instruction is derived from a series of lectures. That is not the purpose of this book. It would be interesting to apply the spirit of operational research to the institution of the university lecture. What purpose are lectures supposed to achieve, and to what extent do they in fact achieve it? For several years now thoughtful teachers have raised this question. Stephen Potter in his book *The Muse in Chains* maintains that universities have simply not realized that printing has been invented, and that the lecture has survived from the time when the lecturer had a handwritten copy of Aristotle which he read at dictation speed in order that each student might possess a similar manuscript. Professor Crowther in New Zealand used to maintain that precise communication was possible only through the written or printed word. When we speak, we improvise. When we write, we examine what we have written and amend it many times until it expresses truly what we wish to say. It was with such remarks in mind that this book was composed. The printed material was handed to the students to read. In the class periods and tutorials, students raised difficulties and these were discussed – between students themselves, between students and graduate assistants, and between students and the ‘lecturer’. It was in such discussions that the most effective learning was done. A clear explanation is a useful starting point, but it is hardly ever the end. Students are liable to misinterpret or fail to understand the explanation in the most individual and unpredictable ways. It is only through discussion that such misunderstandings can be discovered and cleared up.

September 1971

W. W. S.