

# Contents

<i>Preface</i>	<i>page</i> vii
<b>1</b> Introduction and overview	1
<b>2</b> The Burgers equation	6
2.1 <i>Introduction</i>	6
2.2 <i>Derivation of Burgers' equation</i>	9
2.3 <i>Historical background and transformations</i>	15
2.4 <i>The pure initial value problem</i>	21
2.5 <i>Stationary solutions and shock structure</i>	23
2.6 <i>Single hump solution</i>	25
2.7 <i>Planar <math>N</math> wave</i>	28
2.8 <i>Periodic initial conditions</i>	31
2.9 <i>An equivalent boundary value problem</i>	34
2.10 <i>Viscosity method and Burgers' equation</i>	38
<b>3</b> Generalised Burgers equations	40
3.1 <i>Introduction</i>	40
3.2 <i>Generalised Burgers equation with damping</i>	44
3.3 <i>Derivation of the non-planar Burgers equation</i>	51
3.4 <i>Singular perturbation solution for non-planar <math>N</math> waves</i>	61
3.5 <i>The periodic plane piston problem</i>	66
3.6 <i>Singular perturbation analysis of the periodic piston problem</i>	72
3.7 <i>An alternative approach to non-planar <math>N</math> waves</i>	76
3.8 <i>Generalised Burgers equations and their transformations</i>	80
3.9 <i>The inverse transformation</i>	88
3.10 <i>The inhomogeneous Burgers equation</i>	92
<b>4</b> Self-similar solutions as intermediate asymptotics for nonlinear diffusion equations	98
4.1 <i>The nature of self-similar solutions</i>	98
4.2 <i>Fisher's equation</i>	103
4.3 <i>A nonlinear heat equation</i>	114
4.4 <i>Asymptotic behaviour of solutions – intermediate asymptotics</i>	120
4.5 <i>A nonlinear diffusion problem arising in plasma physics</i>	127
4.6 <i>The non-planar Burgers equation</i>	137
4.7 <i>Stability of the self-similar solution of the cylindrically symmetric Burgers equation – another intermediate asymptotic</i>	140

vi	<i>Contents</i>	
	4.8 <i>The linear (similarity) solution as an intermediate asymptotic – the super-cylindrical Burgers equation</i>	147
	4.9 <i>Other generalised Burgers equations</i>	150
	<b>5 Numerical solution of nonlinear diffusion equations</b>	<b>153</b>
	5.1 <i>Introduction</i>	153
	5.2 <i>Implicit finite difference schemes</i>	154
	5.3 <i>Pseudo-spectral numerical scheme</i>	158
	5.4 <i>Accurate space differencing or pseudo-spectral method for a scalar convective equation</i>	159
	5.5 <i>Solution of Fisher's equation by the pseudo-spectral approach</i>	164
	5.6 <i>Non-planar N wave solution by the implicit and pseudo-spectral finite difference approaches</i>	172
	5.7 <i>Generalised Burgers equation with damping</i>	198
	5.8 <i>Non-planar Burgers equation with general nonlinearity</i>	225
	<i>References</i>	231
	<i>Author index</i>	241
	<i>Subject index</i>	244