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TOPOLOGICAL SPACES, NORMALITY AND COMPACTNESS

1 Topological spaces

In this section the definitions and results from point-set topology with which we shall assume familiarity are listed. In addition to making clear the prerequisites for reading the book, the purpose is to establish what forms of concepts, about which there is no general agreement, will be used, and to introduce notation and terminology. In addition to the topics covered in the brief discussion of this section, some acquaintance will be assumed with the conditions of normality and compactness. Later sections of this chapter are devoted to these concepts. We begin by recalling some definitions from the theory of ordered sets.

A *quasi-order* on a set X is a binary relation $\leqslant$ on X which is reflexive and transitive. Thus $x \leqslant x$ for every x, and if $x \leqslant y$ and $y \leqslant z$ then $x \leqslant z$. A *quasi-ordered set* is a set together with a quasi-order on the set. A quasi-ordered set X is said to be a *directed set* if for each pair x, y of elements of X there exists z in X such that $x \leqslant z$ and $y \leqslant z$. A *partial order* on a set is a quasi-order on the set which is anti-symmetric in the sense that $x = y$ if $x \leqslant y$ and $y \leqslant x$. A *partially ordered set* is a set together with a partial order on the set. Let X be a partially ordered set with partial order $\leqslant$. An element x of X is said to be *maximal* if there exists no element y distinct from x such that $x \leqslant y$. Similarly x is said to be *minimal* if there exists no y distinct from x such that $y \leqslant x$. If A is a subset of X, then an *upper bound* of A is an element x such that $a \leqslant x$ for every a in A. The *least upper bound* or *supremum* of A is an upper bound x of A such that $x \leqslant y$ for every upper bound y of A. We shall denote the least upper bound of A by $\sup A$ when it exists. The definitions of *lower bound* and *greatest lower bound* or *infimum* are similar. The notation $\inf A$ is used for the greatest lower bound of A when it exists. If X has a lower bound it is called the least element of X and an upper bound for X, if it exists, is called the greatest element of X. The least and greatest elements of X, when they exist, are called *universal bounds* of X. A partially ordered set in which each finite subset has a least upper bound and a greatest lower bound is called a *lattice*. If x and y are elements of a lattice, we write

$$x \vee y = \sup\{x, y\} \quad \text{and} \quad x \wedge y = \inf\{x, y\}.$$

A lattice is called a *distributive lattice* if the identitites

$$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z),$$

$$x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z),$$

hold. A partial order $\leqslant$ on a set X is called a *linear order* if for each pair x, y of elements of X either $x \leqslant y$ or $y \leqslant x$ holds. A *linearly ordered set* is a set together with a linear order on the set. Each subset of a partially ordered set has an induced order; a subset of a partially ordered set in which the induced order is a linear order is called a *chain*. A linearly ordered set is said to be *order-complete* if each non-empty subset has a least upper bound and a greatest lower bound. If $\leqslant$ is a linear order on a set X let us introduce a relation $<$ on X by putting $x < y$ if $x \leqslant y$ and $x \neq y$. If $a, b \in X$ and $a < b$ let us put

$$(a, b) = \{x \in X \mid a < x < b\},$$

$$(a, b] = \{x \in X \mid a < x \leqslant b\},$$

$$[a, b) = \{x \in X \mid a \leqslant x < b\},$$

$$[a, b] = \{x \in X \mid a \leqslant x \leqslant b\}.$$

This notation will always be used for linearly ordered sets. A subset of a linearly ordered set of any one of the above four forms is called an *interval*. A linearly ordered set is *well-ordered* if each non-empty subset has a least element. Thus if A is a non-empty subset of a well-ordered set, there exists x in A such that $x \leqslant y$ if $y \in A$. We shall frequently use the equivalent form of the axiom of choice that any set can be well-ordered. It follows that any set can be well-ordered so that it has a greatest element. [For if X is a well-ordered set with order $\leqslant$, and x is the least element of X, a new linear order $\leqslant'$ can be defined on X so that $y \leqslant' z$ if and only if $y \leqslant z$ if y and z are distinct from x, and $y \leqslant' x$ for all y. With the linear order $\leqslant'$, X is well-ordered and x is the greatest element of X.] We shall also use *Zorn's lemma* which states that if each chain in a partially ordered set has an upper bound then the set contains a maximal element, and the Kuratowski lemma which states that each chain in a partially ordered set is contained in a maximal chain. Familiarity will be assumed with ordinal numbers and with proof and construction by transfinite induction. Some knowledge of cardinal numbers and their arithmetic will be assumed. The cardinal number of a set X will be denoted by $|X|$. The set of positive integers will be denoted by $\mathbf{N}$. The cardinal number $|\mathbf{N}|$ is the first infinite cardinal, denoted by $\aleph_0$. The first uncountable cardinal is denoted by $\aleph_1$. The cardinal number of the set $\mathbf{R}$ of real numbers is denoted by $\mathbf{c}$.

A *topological space* is a set X and a set $\mathscr{T}$ of subsets of X, called the *open sets* of X, such that $\mathscr{T}$ contains $\varnothing$ and X, and is closed under the formation of finite intersections and arbitrary unions. The set $\mathscr{T}$ of open sets is called the *topology* of the topological space. Thus a topological space is a pair $(X, \mathscr{T})$. If no confusion can arise, the topology $\mathscr{T}$ is not mentioned and we speak of the 'topological space X'. If X is a set and $\mathscr{T}$ is the set of all subsets of X then $\mathscr{T}$ is a topology called the *discrete topology*. The set of subsets of X which consists of $\varnothing$ and X only is a topology called the *trivial topology*. A subset N of a topological space X is a *neighbourhood* of a point x of X if there exists an open set U such that $x \in U \subset N$. Similarly N is a neighbourhood of a subset A of X if there exists an open set U such that $A \subset U \subset N$. A set U is open if and only if U contains a neighbourhood of each of its points. A *base* for the topology of a space X is a set $\mathscr{B}$ of open sets such that for every point x of X and every neighbourhood N of x there exists B in $\mathscr{B}$ such that $x \in B \subset N$. Equivalently $\mathscr{B}$ is a base if each open set is a union of members of $\mathscr{B}$. A *subbase* for a topology is a set $\mathscr{S}$ of subsets such that the set of all finite intersections of members of $\mathscr{S}$ is a base for the topology. The *interior* A° of a subset A of a topological space is the largest open set contained in A. A set U is open if and only if $U = U^\circ$. For subsets A and B we have

$$(A \cap B)^\circ = A^\circ \cap B^\circ.$$

A subset A of a topological space X is said to be *closed* if its complement $X \backslash A$ is open. Clearly any intersection of closed sets is a closed set and any finite union of closed sets is a closed set. In a topological space X, the sets $\varnothing, X$ are closed, and so are open-and-closed sets. The topological space X is said to be *connected* if there are no open-and-closed sets other than $\varnothing$ and X. The *closure* $\bar{A}$ of a subset A of a topological space is the smallest closed set containing A. A point x belongs to $\bar{A}$ if and only if $N \cap A \neq \varnothing$ for every neighbourhood N of x. For typographical convenience, the closure of A will sometimes be denoted by $(A)^-$. A set A is closed if and only if $A = \bar{A}$. If A is a subset of a topological space X then

$$X \backslash \bar{A} = (X \backslash A)^{\circ}.$$

For subsets A and B we have

$$(A \cup B)^- = \bar{A} \cup \bar{B}.$$

If A is a subset of a topological space X, then a point x of X is said to be an *accumulation point* of A if every neighbourhood of x contains a point of A distinct from x. Clearly $\bar{A}$ is the union of A and the set of accumulation points of A. A subset A is *dense* in a topological space X if $\bar{A} = X$. If A is a subset of a topological space X and V is an open set of X, then $V \cap \bar{A} \subset (V \cap A)^-$. [For if $x \in V \cap \bar{A}$ and N is a neighbourhood of x, then $N \cap V$ is a neighbourhood of x so that

$$N \cap V \cap A \neq \varnothing$$

and hence $x \in (V \cap A)^-$.] It follows that if A is a dense subset and V is an open set then $V \subset (V \cap A)^-$, so that $\bar{V} \subset (V \cap A)^-$ and hence $\bar{V} = (V \cap A)^-$. A *covering* of a topological space is a family $\mathscr{A} = \{A_\lambda\}_{\lambda \in \Lambda}$ of subsets such that

$$\bigcup_{\lambda \in \Lambda} A_\lambda = X.$$

If each set A_λ is open, then $\mathscr{A}$ is called an *open covering*, and if each set A_λ is closed, then $\mathscr{A}$ is called a *closed covering*. A covering $\{B_\gamma\}_{\gamma \in \Gamma}$ is said to be a *refinement* of a covering $\{A_\lambda\}_{\lambda \in \Lambda}$ if for each γ in Γ there exists some λ in Λ such that $B_\gamma \subset A_\lambda$. If Y is a subset of a topological space X, the induced topology for Y consists of the sets of the form $U \cap Y$, where U is an open set of X. The subset Y with the induced topology is a *subspace* of X. If Y is a subspace of X and B is a subset of Y, then the closure of B in Y is $\bar{B} \cap Y$, where $\bar{B}$ is the closure of B in X. A subset Y of a topological space is said to be a connected set if the subspace Y is connected. Since the union of connected sets with a common point is connected, it follows that each point of a topological space is contained in a largest connected set which is called the *component* of the point. Since the closure of a connected set is connected, the components are closed sets. The set of all distinct components in a space forms a partition of the space.

Many examples in dimension theory involve linearly ordered spaces. The interval topology on a linearly ordered set X has a subbase consisting of all sets of the form

$$\{x \in X \mid x > a\} \quad \text{and} \quad \{x \in X \mid x < b\}$$

for a, b in X. A linearly ordered set with the interval topology is called a *linearly ordered space*. If X has neither a least element nor a greatest element, then a base for the interval topology consists of all 'open' intervals (a, b), where a, b are elements of X such that $a < b$. If X has a least element x_0, all intervals $[x_0, b)$ for b in $X\backslash\{x_0\}$ must be added, and if X has a greatest element x_1, all intervals $(a, x_1]$ for a in $X\backslash(\{x_1\}$ must be added. The interval topology on the set $\mathbf{R}$ of real numbers with respect to its natural linear order is the usual topology for $\mathbf{R}$. The closed subspace $[0, 1]$ is the *unit interval* and will usually be denoted by I.

If X and Y are sets and $f: X \to Y$ is a function, then the inverse image of a subset B of Y is the set

$$f^{-1}(B) = \{x \in X \mid f(x) \in B\}.$$

In the case of one-point subsets we change the notation slightly, writing

$$f^{-1}(y) = \{x \in X \mid f(x) = y\}$$

if $y \in Y$. If X and Y are topological spaces, then $f: X \to Y$ is said to be a *continuous function* (mapping or continuous mapping) if $f^{-1}(U)$ is an open set of X for each open set U of Y. We list a number of conditions equivalent to continuity: (a) the inverse image of each member of a subbase for the topology of Y is an open set of X; (b) the inverse image of each closed set of Y is a closed set of X; (c) for every point x of X, the inverse image of every neighbourhood of $f(x)$ is a neighbourhood of x; (d) $f(\bar{A}) \subset (f(A))^-$ for each subset A of X; (e) $(f^{-1}(B))^- \subset f^{-1}(\bar{B})$ for each subset B of Y. The composite of continuous functions is continuous. The continuous real-valued functions on a topological space are of particular importance. Let X be a topological space and let $f, g: X \to \mathbf{R}$ be continuous functions, where $\mathbf{R}$ has the usual topology. The real-valued functions $|f|, f+g, fg$, given by

$$|f|(x) = |f(x)|,$$

$$(f+g)(x) = f(x) + g(x),$$

$$(fg)(x) = f(x)\, g(x),$$

if $x \in X$, are continuous. If $\lambda \in \mathbf{R}$, then the function λf, given by $(\lambda f)(x) = \lambda f(x)$ if $x \in X$, is continuous. The function $h = \max\{f, g\}$ is continuous, where

$$h(x) = \max\{f(x), g(x)\}$$

if $x \in X$. Similarly the function $\min\{f, g\}$ is continuous. If $g(x) \neq 0$ for all x in X we can define a real-valued function f/g by putting

$$(f/g)(x) = f(x)/g(x)$$

if $x \in X$, and f/g is continuous. If $\{f_i\}_{i \in \mathbf{N}}$ is a sequence of continuous real-valued functions on a topological space X, and for each i, $|f_i(x)| \leqslant M_i$ for all x, where the series $\Sigma_{i=1}^{\infty} M_i$ is convergent, then the series $\Sigma_{i=1}^{\infty} f_i(x)$ is absolutely convergent for each x, and we can define a real-valued function f on X by putting

$$f(x) = \sum_{i=1}^{\infty} f_i(x)$$

if $x \in X$, and f is continuous. If X and Y are topological spaces, a bijection $h: X \to Y$ is said to be a *homeomorphism* if both h and h^{-1} are continuous. Equivalently, a continuous function $h: X \to Y$ is a homeomorphism if there exists a continuous function $g: Y \to X$ such that

$$g \circ h = 1_X \quad \text{and} \quad h \circ g = 1_Y,$$

where 1_X is the identity mapping on X given by $1_X(x) = x$ for all x, and similarly 1_Y is the identity mapping on Y. If $h: X \to Y$ is a homeomorphism then h induces a bijective correspondence between the topologies of X and Y. If there exists a homeomorphism between X and Y then X and Y are said to be homeomorphic. The relation of being homeomorphic is an equivalence relation in the class of all topological spaces. A property which, when possessed by a given space, is possessed by all homeomorphic spaces is called a *topological property* or a topological invariant. Clearly each property of a topological space which is defined in terms of open sets and concepts derived from open sets and the notions of set theory is a topological property. A continuous function $f: X \to Y$ is said to be *open* if $f(U)$ is an open set of Y for each open set U of X, and f is said to be *closed* if $f(E)$ is a closed set of Y for each closed set E of X. The following statements about a function h are equivalent: (*a*) h is a homeomorphism; (*b*) h is a continuous open bijection; (*c*) h is a continuous closed bijection. A mapping $f: X \to Y$ is closed if and only if $f(\bar{A}) = (f(A))^-$ for every subset A of X. If X and Y are topological spaces, $f: X \to Y$ is a continuous function and A and B are subspaces of X and Y respectively such that $f(A) \subset B$, then the function $g: A \to B$ given by $g(x) = f(x)$ for each x in A is continuous. The mapping g is said to be given by restriction of f. If f is open and A is open then g is open, and similarly g is closed if f is closed and A is closed. We shall write $f|A$ to denote the mapping of

A into Y given by restriction of f. If B is a subset of Y, then for each subset H of X

$$f(f^{-1}(B) \cap H) = B \cap f(H).$$

Thus the mapping of $f^{-1}(B)$ into B given by restriction of f is open if f is open and closed if f is closed. A continuous function $f: X \to Y$ is said to be an *embedding* of X in Y if the mapping of X onto $f(X)$, given by restriction of f, is a homeomorphism. If there exists an embedding of X in Y, then X is homeomorphic with a subspace of Y and thus X has any topological property which is inherited by subspaces of Y. If A is a subspace of a topological space X, then a continuous function $r: X \to A$ such that $r(x) = x$ for each x in A is called a *retraction* of X onto A. If there exists a retraction of X onto A, then A is said to be a *retract* of X.

Let $\{X_\lambda\}_{\lambda \in \Lambda}$ be a family of topological spaces and let X be the cartesian product $\Pi_{\lambda \in \Lambda} X_\lambda$. The points of X are indexed families $\{x_\lambda\}_{\lambda \in \Lambda}$, where $x_\lambda \in X_\lambda$ for each λ. For λ_0 in Λ, the projection $\pi_{\lambda_0}: X \to X_{\lambda_0}$ is given by $\pi_{\lambda_0}(\{x_\lambda\}) = x_{\lambda_0}$. The set X with the smallest topology such that every projection π_λ is continuous is the *topological product* of the family $\{X_\lambda\}_{\lambda \in \Lambda}$. A base for the product topology consists of all sets of the form $\bigcap_{\lambda \in M} \pi_\lambda^{-1}(V_\lambda)$, where M is a finite subset of Λ and V_λ is an open set of X_λ for λ in M. In fact X has a subbase for its topology which consists of all sets of the form $\pi_\lambda^{-1}(V_\lambda)$, where $\lambda \in \Lambda$ and V_λ is a member of a given subbase for the topology of X_λ. If for each λ, $g_\lambda: Z \to X_\lambda$ is a continuous function, then the unique function $g: Z \to X$ such that $\pi_\lambda \circ g = g_\lambda$, is continuous. It follows that if $\{X_\lambda\}_{\lambda \in \Lambda}$ and $\{Y_\lambda\}_{\lambda \in \Lambda}$ are families of topological spaces with topological products X and Y respectively and projections π_λ and ρ_λ for each λ, and $f_\lambda: X_\lambda \to Y_\lambda$ is a continuous function for each λ, then there exists a unique continuous function $f: X \to Y$ such that $\rho_\lambda \circ f = f_\lambda \circ \pi_\lambda$ for every λ. We shall write $f = \Pi_{\lambda \in \Lambda} f_\lambda$. If $\{X_\lambda\}_{\lambda \in \Lambda}$ is a family of topological spaces with topological product X and A_λ is a subspace of X_λ for each λ, then the topology induced on $\Pi_{\lambda \in \Lambda} A_\lambda$ as a subspace of X coincides with the product topology. The *topological sum* of a family $\{X_\lambda\}_{\lambda \in \Lambda}$ of topological spaces is the disjoint union X of the sets X_λ with the largest topology such that the inclusion of each X_λ in X is continuous. The open sets of the topological sum are the disjoint unions of families $\{U_\lambda\}_{\lambda \in \Lambda}$, where U_λ is an open set of X_λ for each λ. Each space X_λ is embedded in X as an open-and-closed subspace. If X is a topological space, Y is a set and $f: X \to Y$ is a surjection, then the *identification topology* on Y with respect to f is the largest topology on Y such that

f is continuous. Thus a subset V of Y is an open set if and only if $f^{-1}(V)$ is an open set of X. If X and Y are topological spaces and $f: X \to Y$ is a continuous surjection, then f is called an *identification mapping* if Y has the identification topology with respect to f. Any continuous open surjection and any continuous closed surjection is an identification mapping. If $f: X \to Y$ is an identification mapping and $h: X \to Z$ is a continuous function such that h is constant on $f^{-1}(y)$ for each y in Y, then the function $g: Y \to Z$ such that $g \circ f = h$ is continuous. If R is an equivalence relation in a topological space X, then the *quotient space* of X with respect to R is the set Y of equivalence classes of X with respect to R with the identification topology with respect to the surjection $f: X \to Y$ such that $x \in f(x)$ for each x in X. If $f: X \to Y$ is an identification mapping, then Y is homeomorphic with the quotient space of X with respect to the equivalence relation R defined by xRx' if and only if $f(x) = f(x')$ for x, x' in X.

Next we introduce some 'separation axioms'. In this work, a topological space will not be assumed to satisfy any separation axiom unless it is explicitly stated. The most important separation property for dimension theory is normality which will be studied in § 3. Consider the following conditions on a topological space:

(T_0) For every pair of distinct points there exists a neighbourhood of one of them which does not contain the other.

(T_1) For every pair of distinct points there exists a neighbourhood of each of them which does not contain the other.

(T_2) For every pair of distinct points x and y there exist a neighbourhood of x and a neighbourhood of y which are disjoint.

A topological space is said to be a *T_i-space*, where $i = 0, 1, 2$, if it satisfies the axiom (T_i) above. Topological spaces which satisfy (T_2) are also called *Hausdorff spaces*. Clearly every T_2-space is a T_1-space and every T_1-space is a T_0-space. The reverse implications do not hold and there exist spaces which do not satisfy the axiom (T_0). A topological space is a T_1-space if and only if each set which consists of a single point is closed. A topological space is said to be a *regular space* if for each point x and each closed set F not containing x there exist disjoint open sets U and V such that $x \in U$ and $F \subset V$. Equivalently, a topological space is a regular space if each neighbourhood of each point contains a closed neighbourhood of the point. Hausdorff spaces are not necessarily regular and regular spaces are not necessarily T_0-spaces. A regular T_0-space, however, is a Hausdorff space. A regular

T_0-space is said to be a T_3-space. The separation properties so far considered are preserved under the operations of taking subspaces and forming topological products.

A *pseudo-metric* on a set X is a function $d: X \times X \to \mathbf{R}$ such that for all x, y, z in X: (i) $d(x,y) = d(y,x)$; (ii) $d(x,z) \leqslant d(x,y) + d(y,z)$; (iii) $d(x,x) = 0$. If d is a pseudo-metric on a set X, then $d(x,y) \geqslant 0$ for all x, y in X. If d is a pseudo-metric on a set X and d satisfies in addition the condition that $d(x,y) = 0$ implies $x = y$, then d is a *metric* on X. A *pseudo-metric space* is a pair (X, d), where X is a set and d is a pseudo-metric on X. If d is a metric then (X, d) is a *metric space*. If (X, d) is a pseudo-metric space then there is a topology induced by d on X. For each point x of X and each positive real number r let

$$B_r(X) = \{y \in X \mid d(x,y) < r\}.$$

The family of sets $B_r(x)$ for $r > 0$ and x in X is the base for a topology on X. This topology is called the pseudo-metric topology on X or the topology on X induced by d. We shall call $B_r(x)$ the *open ball* with centre x and radius r. The notation $B_r(x)$ for an open ball will be used without further explanation whenever a pseudo-metric or metric space is discussed. If (X, d) is a pseudo-metric space, $x \in X$ and A is a non-empty subset of X, then the set of real numbers $\{d(x,y) \mid y \in A\}$ is non-empty and bounded below. We define

$$d(x, A) = \inf_{y \in A} d(x,y).$$

The real-valued function on X which associates with each point x the real number $d(x, A)$ is continuous, and

$$\bar{A} = \{x \in X \mid d(x, A) = 0\},$$

$$A^0 = \{x \in X \mid d(x, X \backslash A) > 0\}.$$

For each positive real number the set

$$B_r(A) = \{x \in X \mid d(x, A) < r\}$$

is open. If E and F are non-empty subsets of a pseudo-metric space, then the set of real numbers $\{d(x,y) \mid x \in E, y \in F\}$ is bounded below. We define

$$d(E, F) = \inf\{d(x,y) \mid x \in E, y \in F\}.$$

If $d(E, F) > 0$, then $\bar{E}$ and $\bar{F}$ are disjoint. A pseudo-metric space (X, d) is a regular space and (X, d) is a T_3-space if and only if d is a metric. A topological space X is said to be *pseudo-metrizable* if there exists a pseudo-metric d on X such that the topology induced by d is the topology of X. Similar X is *metrizable* if its topology is induced by a metric. A pseudo-metrizable space is metrizable if it is a T_0-space. The pseudo-metrizable and metrizable spaces will be characterized in several ways in Chapter 2. The space $\mathbf{R}$ of real numbers is metrizable, its topology being induced by the metric d given by $d(x, y) = |x-y|$ for x, y in $\mathbf{R}$.

Let n be a positive integer and consider the cartesian product $\mathbf{R}^n$ of n copies of the set $\mathbf{R}$ of real numbers. If $x = (x_1, \ldots, x_n)$ is an element of $\mathbf{R}^n$, let

$$\|x\| = \sqrt{\left(\sum_{i=1}^{n} x_i^2\right)}.$$

In the case $n = 1$, the usual notation $|x|$ instead of $\|x\|$ will be used. Euclidean n-dimensional space is the set $\mathbf{R}^n$ with the topology induced by the metric d, where

$$d(x, y) = \|x-y\|$$

for x, y in $\mathbf{R}^n$. Thus 1-dimensional Euclidean space is the usual space for real numbers. For each positive integer n let

$$S^{n-1} = \{x \in \mathbf{R}^n \mid \|x\| = 1\},$$

$$E^n = \{x \in \mathbf{R}^n \mid \|x\| \leqslant 1\}.$$

We shall call S^{n-1} and E^n the *unit sphere* and the *closed unit ball* respectively. The origin $(0, \ldots, 0)$ in $\mathbf{R}^n$ will be denoted by $\mathbf{0}$. The following result is an important part of the proof that Euclidean n-dimensional space is indeed n-dimensional in the sense of the dimension theories to be studied in this book.

Brouwer Theorem. *For each positive integer n, S^{n-1} is not a retract of E^n.*

2 Local finiteness, the weak topology and the weight of topological spaces

If $\{A_\lambda\}_{\lambda \in \Lambda}$ is a family of closed sets of a space X, it is not necessarily true that $\bigcup_{\lambda \in \Lambda} A_\lambda$ is a closed set. It is, of course, true if Λ is finite. We can extend this result to families which are 'locally finite'. The concept of local finiteness proves to be important throughout general topology.