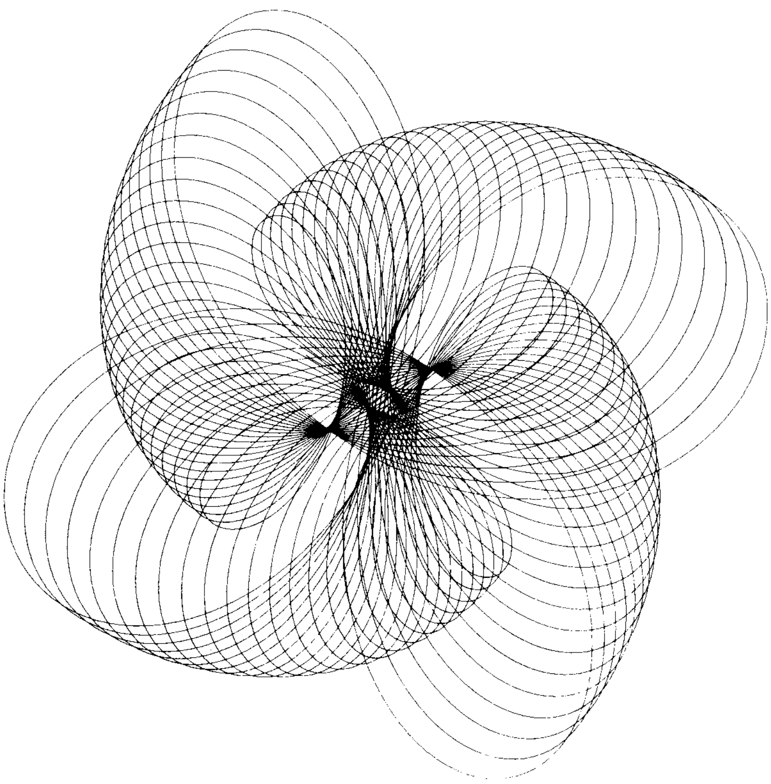
GEOMETRIC SYMMETRY

TO HILDA AND ANNA

All ornament should be based upon a
geometrical construction.
Owen Jones: *The Grammar of Ornament*

Geometric Symmetry



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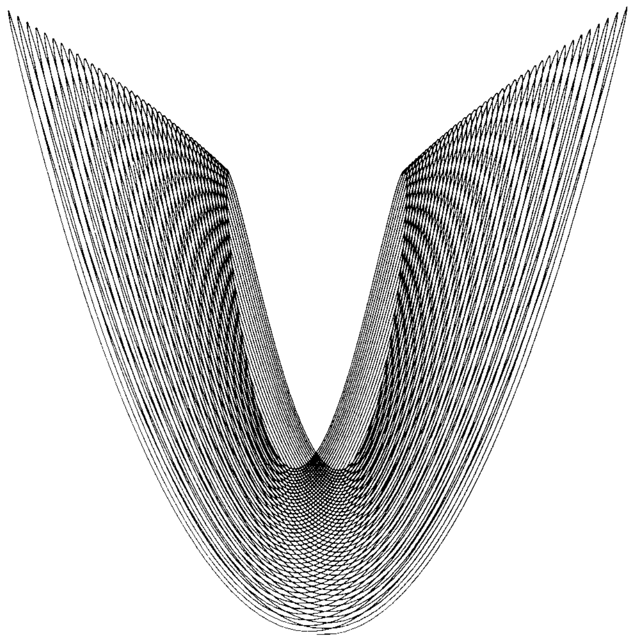

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Preface

Symmetry is of interest in two ways, artistic and mathematical. It underlies much scientific thought, playing an important role in chemistry and atomic physics, and a dominant one in crystallography. It is important in architectural and engineering design and particularly in the decorative arts. Yet the literature available is comparatively sparse. Mathematically it has been extensively treated by continental authors, particularly, in recent times, in Russia. Books on crystallography naturally devote considerable attention to it, giving at least a descriptive account, and the *International Tables for X-ray Crystallography* (vol. 1) is an invaluable work of reference. For the general reader there is Weyl's *Symmetry*, a delightful and stimulating book, and more recently Rosen's *Symmetry Discovered*: but beyond these there is little more than a few short sections in mathematical books that deal mainly with other topics.

In this book we attempt to provide a fairly comprehensive account of symmetry in a form acceptable to readers without much mathematical knowledge or experience who nevertheless want to understand the basic principles of the subject. It is hoped that it will be found useful in school and other libraries and as preliminary reading for students of crystallography. The treatment is geometrical, which should appeal to art students and to readers whose mathematical interests are that way inclined. It is also hoped that the full enumeration of symmetry types will make it useful for reference purposes.

Part I is largely descriptive and is written with the non-mathematical reader in mind. It gives a general account of the subject and indicates how the ideas may be applied to the construction of new designs from very simple elements. Part II is more mathematical, but only the most elementary knowledge of geometry is assumed. The more systematic treatment in this part makes it possible to enumerate and classify the symmetry groups of each kind, using throughout the convenient notation that has been evolved by crystallographers during the last 50 years and is now accepted internationally.

The results in this book are not new, though some of them have been available only in a form not readily accessible. The approach adopted, however, stems from a basic reappraisal of the subject from the geometrical point of view. Our intention, in dividing the book into two parts, has been firstly to reveal the simplicity and beauty of the underlying ideas and then to show that the geometrical approach can lead to a consistent mathematical development of the subject.

Our thanks are due to Dr A.G. Howson, who read the first draft, and to the Press for their ready and patient cooperation and for tackling so successfully the problems that arose in producing the book.

E.H.L.
R.H.M.

November 1977

Historical note

In 1611 Kepler wrote a little monograph *Strena seu de Nive Sexangula* (A New Year Gift: On Hexagonal Snow) in the course of which he considered the packing together of circles in a plane and spheres in space. This was printed, but attracted little attention. The idea that external form might depend on internal structure appeared again, however, as soon as crystals began to be seriously studied. Hooke (*Micrographia*, 1665) thought they might be built up of spheroids. Bartholinus (1669) studied calcite (Iceland spar), noticing the double refraction produced by the crystals and the rhomboidal cleavage planes. He measured the angles between the faces. Nicolaus Steno studied the geometrical forms of crystals and noted the constancy of the angles. Huygens (*Traité de la Lumière*, 1690) tried to explain the calcite phenomena by supposing the crystals to be built up of closely packed spheroids. Bergman (1773) and Haüy (1782) thought in terms of a structure of brick-like elements (*molécules soustractive*, as they were called), but these ideas were slow to develop because of the lack of an adequate atomic theory. Seeber (1824) supposed molecules to be spaced out at intervals, but he was ahead of his time and only in 1879 was the idea revived, by Sohncke. Meantime Hessel (1830) and Axel (1867) had classified crystals into 32 classes and Bravais (1850) had described the 14 space lattices known by his name. In 1890 the 230 space groups were enumerated by Fedorov, and independently by Schoenflies.

The results for line groups and plane groups were implicit in this, or at least could be easily derived. But the interest in symmetry was so entirely concentrated on its application to crystallography that such simple matters as the 7 'frieze' groups and the 17 'wallpaper' groups were not specifically dealt with until much later, notably by Polya[1] and Niggli[1] in 1924 and by Speiser[2] in 1927. Speiser also enumerated the 75 crystallographic line groups in three dimensions and the 31 'ribbon' groups. The 80 'layer' groups were dealt with two years later by Alexander and Herrmann,[3] and also by Weber.[3] In 1930 Heesch[4] and Shubnikov[4] both wrote on the symmetry of continuous and semi-continuous figures.

Mathematically it might have seemed that this was the end of the road, but there was to be a further development. In 1930 Heesch[4] suggested the possibility of four-dimensional symmetry groups in three-dimensional space. The extra dimension could be represented by a change of colour. H.J. Woods[5] (1935) discussed the same possibility under the name 'Counterchange symmetry' and listed the 17 types of particoloured friezes ('counterchange borders'). The idea was taken up by Shubnikov in 1945 and developed by him in *Symmetry and Antisymmetry of Finite Figures* (1951). This dealt with point groups only. Cochran[6] in 1952 listed the 46 particoloured plane groups in two dimensions, deriving them from the 80 layer groups enumerated earlier by

Alexander and Hermann. In 1953 Zamorzaev enumerated the 1651 dichromatic space groups, calling them the *Shubnikov groups*. These were obtained independently in 1955 by Belov, with Nerovina and Smirnova.

As the term *antisymmetry* implies, Shubnikov's extra dimension consisted of a polarity and could be represented by two colours. The idea was later extended to three or more colours by Belov and others.

The study of symmetry, like that of some other branches of mathematics, has been confused by a multiplicity of notations. Fortunately the International Union of Crystallography has established a standard notation, now widely accepted, for the uncoloured groups. For dichromatic change a simple extension, first used by H.J. Woods,[5] has been generally adopted, but there is as yet no recognized notation for polychromatic groups.

References:

1 *Zeitschrift für Kristallografia*, **60** (1924)
2 Speiser: *Die Theorie der Gruppen* (1927)
3 *Zeit.f.Krist.* **70** (1929)
4 *Zeit.f.Krist.* **73** (1930)
5 *Journal of Textile Inst.* **26** (1935)
6 *Acta Crystallagr.* **5** (1952)