

Cambridge University Press  
 978-0-521-09228-9 - General Homogeneous Coordinates in Space of Three  
 Dimensions

E. A. Maxwell

Table of Content

[More information](#)

## CONTENTS

	PAGE
PREFACE	xi
INTRODUCTION	xiii
CHAPTER I. The point, the straight line and the plane	1
1. Knowledge assumed. 2. The homogeneous coordinates. 3. The symbol of a point. 4. The straight line. 5. The plane. 6. The intersection of two planes. 7. The syzygy connecting the symbols of five given points. 8. The transversal from a given point to two given skew lines. 9. The tetrahedron of reference. 10. The unit point and the unit plane. 11. Duality. 12. Plane coordinates. 13. The equation of a point. 14. Transformation of coordinates. 15. Cross-ratio properties. 16. A transversal theorem for a tetrahedron (Von Staudt's theorem).	
<i>Illustration 1.</i> Theorem of Desargues.	
<i>Illustration 2.</i> The polar plane of a point with respect to a tetrahedron.	
<i>Illustration 3.</i> Möbius's tetrahedra.	
<i>Illustration 4.</i> Harmonic inversion.	
<i>Illustration 5.</i> Desmic tetrahedra.	
Examples I. Miscellaneous Examples I.	
Appendix to Chapter I.	25
CHAPTER II. The quadric surface	32
1. Introduction. 2. The expression of a quadratic form as a 'sum of squares'. 3. A simple form for the equation of a quadric.	
<i>Illustration 1.</i> A plane through $T$ cuts the cone in the points of two straight lines, which may 'coincide'.	
4. Notation. 5. Joachimstal's equation. 6. Tangency. 7. Conjugacy. 8. The condition for a plane to touch a quadric. 9. Duality. 10. The equation of a quadric envelope as a 'sum of squares'. 11. Tangency and conjugacy for a quadric envelope. 12. Quadric locus and quadric envelope. 13. Reciprocation. 14. The reciprocal of a straight line. 15. Analytical treatment of reciprocation. 16. The properties of a cone. 17. Self-polar tetrahedron.	
<i>Illustration 2.</i> The poles of the faces of the tetrahedron of reference with respect to a quadric envelope.	
<i>Illustration 3.</i> $S$ is a given quadric, $\pi$ an arbitrary plane and $P_1$ an arbitrary point. To prove that the cone projecting the conic $C \equiv (S\pi)$ from $P_1$ meets $S$ again in a conic, and that the equation of the plane of the conic is $S_{11}\pi - 2\pi_1S_1 = 0$ .	
Examples II. Miscellaneous Examples II.	

Cambridge University Press

978-0-521-09228-9 - General Homogeneous Coordinates in Space of Three Dimensions

E. A. Maxwell

Table of Content

[More information](#)

viii

CONTENTS

**CHAPTER III. The generators of a quadric surface** 53

1. A simple form for the equation of a quadric. 2. The two systems of generators. 3. The tangent plane at a point. 4. The projective generation of a quadric surface. 5. Polar lines. 6. Conjugate lines.

*Illustration 1.* To find the conditions that the lines  $x = 0, t = 0$  and  $y = 0, z = 0$  should be polar lines with respect to the general quadric  $S$ .

*Illustration 2.* To find the condition that the lines  $x = 0, t = 0$  and  $y = 0, z = 0$  should be conjugate lines with respect to the quadric  $S$ .

*Illustration 3.* Alternative proof of Von Staudt's theorem (Chapter I, § 16).

*Illustration 4.* The reciprocation of one quadric into another.

Examples III. Miscellaneous Examples III.

**CHAPTER IV. Line geometry** 65

1. Preliminary remarks. 2. The numbers  $l, m, n, l', m', n'$ . 3. The coordinates of a line. 4. An algebraic theorem and some important deductions. 5. The line as the intersection of two planes. 6. To find the coordinates of the point in which a given line meets a given plane. 7. The linear complex. 8. The linear congruence. 9. The regulus. 10. The quadric whose generators of one system are the lines common to three linear complexes. 11. The lines common to four linear complexes. 12. Polar lines with respect to a quadric. 13. The quadratic complex. 14. The tetrahedral complex.

*Illustration.* The poles, with respect to a quadric, of the faces  $YZT, ZXT, XYT, XYZ$  of a tetrahedron  $XYZT$  are  $X', Y', Z', T'$  respectively. To prove that the lines  $XX', YY', ZZ', TT'$  are generators of one system on a quadric.

Examples IV. Miscellaneous Examples IV.

**CHAPTER V. The twisted cubic** 82

1. Definition and first properties. 2. The standard parametric form. 3. Properties deduced from the parametric form. 4. Tangent line and osculating plane. 5. The quadrics through the curve. 6. Involutions on the twisted cubic. 7. The line-coordinates of chords and tangents. 8. The cubic developable.

Examples V. Miscellaneous Examples V.

**CHAPTER VI. Systems of quadrics** 98

1. Preliminary remarks. 2. Polar properties of a pencil of quadrics. 3. The cones of a pencil and the standard form of equation. 4. Other types of pencil. 5. Equality of coefficients in the standard form. 6. The pencil of quadrics defined by two conics meeting in two distinct points. 7. The pencil of quadrics touching along a given conic. 8. The pencil of quadrics with four common generators. 9. Tangential pencils. 10. Nets of quadrics; associated points.

*Illustration.* Given that  $XYZT, ABCD$  are two tetrahedra with the property that  $A, B, C, D$  lie respectively in the planes  $YZT, ZXT, XYT, XYZ$  while  $X, Y, Z$  lie in the planes  $BCD, CAD, ABD$ . To prove that  $T$  lies in the plane  $ABC$ .

Miscellaneous Examples VI.

Cambridge University Press  
 978-0-521-09228-9 - General Homogeneous Coordinates in Space of Three  
 Dimensions

E. A. Maxwell

Table of Content

[More information](#)

CONTENTS		ix
CHAPTER VII. Applications to Euclidean Geometry		114
1. Homogeneous Cartesian coordinates; the plane at infinity. 2. Parallel straight lines. 3. Parallel planes. 4. Length or distance.		
<i>Illustration 1.</i> The centroid of a tetrahedron.		
5. First properties of quadrics. 6. The sphere. 7. The right angle.		
<i>Illustration 2.</i> The orthogonal tetrahedron.		
<i>Illustration 3.</i> The configuration of the cube.		
8. Further properties of spheres. 9. Further properties of quadrics. 10. The circular sections; quadrics of revolution. 11. Confocal quadrics and normals to quadrics; preliminary projective properties. 12. Confocal quadrics and normals to quadrics; metrical properties.		
Miscellaneous Examples VII.		
CHAPTER VIII. The use of matrices		134
<i>Section I. Matrices</i>		
1. First properties. 2. Multiplication by a scalar. 3. Scalar linear combinations of matrices. 4. Matrix multiplication. 5. The transpose of a matrix. 6. The inverse matrix.		
<i>Section II. Geometrical Applications</i>		
7. Coordinates and the plane.		
<i>Illustration 1.</i> To find the coordinates of the point where the line joining the points $p, q$ meets the plane $u$ .		
8. Transformation of coordinates. 9. The bilinear form. 10. Polar theory of the quadric.		
<i>Illustration 2.</i> Worked example.		
<i>Illustration 3.</i> Worked example.		
11. Line-coordinates.		
<i>Illustration 4.</i> To prove that the equation of the quadric generated by the lines common to three non-singular linear complexes, of matrices $a, b, c$ , may be expressed in the form $(x'qx) = 0$ , where $q \equiv cb^{-1}a - ab^{-1}c$ .		
<i>Illustration 5.</i> Worked example.		
GENERAL EXAMPLES		160
ANSWERS TO EXAMPLES		166
INDEX		167