GENERAL HOMOGENEOUS COORDINATES IN SPACE OF THREE DIMENSIONS

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DEDICATED

TO

MY WIFE

GRETA LOUISE MAXWELL

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PREFACE

I AM deeply indebted to two lecturers in the University of Cambridge for their help and advice in the preparation of this book. The manuscript was read by Dr S. Wylie and the proofs by Dr J. A. Todd, F.R.S., and the adoption of their suggestions has added considerably to the clarity and accuracy of the text.

I should also like to record my thanks to a number of pupils, whose corrections I was happy to receive.

To the staff of the Cambridge University Press I again express my appreciation of their printing and of the courteous help which I have always received.

E.A.M.

QUEENS' COLLEGE
CAMBRIDGE

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INTRODUCTION

THE purpose of this book is at once modest and ambitious, namely, to provide a *short* introduction to algebraic geometry in space of three dimensions, to make clear its spirit, and to prepare the way for deeper study. I have in mind a reader who has just read my book on homogeneous coordinates in a plane (to which this stands as a second volume) and is in the early stages of his second-year work at the University. I have also in mind a class of reader who has read further in mathematics generally, but has found the existing detailed accounts of this work too full or too specialised for his own needs; I hope that such a reader will find here a temptation to get to grips with the subject.

In spite of the existence of a large number of text-books on the geometry of space of three dimensions, I think it is true that there are few which deal with the subject in the essential spirit of projective geometry. The two accounts which seem to me to be of greatest importance for further study are, first, a well-established authority, the *Principles of Geometry*, Vol. III, by Prof. H. F. Baker, and, secondly, an important recent work, *Projective and Analytical Geometry*, by Dr J. A. Todd which for the first time (I think) establishes in readily available form the synthesis of projective geometry with modern matrix algebra. The aim of the present book will be fulfilled if it encourages the reader to turn to these two accounts and, perhaps, helps him a little along the way.

In place of the elementary examples which usually appear throughout each chapter in a book of this kind, I have included a fairly large number of *Theorem-examples*; these are almost entirely standard results, which should be known, and which follow directly from the preceding work. *The conscientious solution of these examples is an essential part of the reading of this book*, and I have added short hints which should remove any possible difficulty.

A reader at this stage will almost certainly be under a teacher, and may well have to call upon him for help in the solution of some of the Miscellaneous Examples at the ends of the chapters. These have appeared in examination papers at what we now seem to call 'the highest level', and are not all easy. They are taken from Preliminary (P.) or Mathematical Tripos (M.T.) papers set in the University of Cambridge, or from Honours papers set in the University of London (L.), and I am grateful for permission to use them. In the earlier chapters I have preceded the Miscellaneous Examples with some numerical work to give practice in manipulation.

There are occasional references to my earlier book, which for brevity I denote by the letter M.

Finally, a word should be said about the last chapter. I have thought it right to bring the reader to the threshold of the methods now in use, but appreciate that algebraic equipment may vary considerably. I have therefore given a brief self-contained summary of the elements of matrix algebra, and then demonstrated how it can be applied. The reader will readily appreciate the mental economy which the introduction of matrices provides. As far as I am able to judge, the paragraph on line-coordinates in this chapter contains several results which simplify existing treatments.

E. A. M.