THEORY OF LAMINAR FLAMES

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PREFACE

Existing combustion books are primarily phenomenological in the sense that explanation, where provided, is usually set in an intuitive framework; when mathematical modeling is employed it is often obscured by *ad hoc* irrational approximation, the emphasis being on the explanation of existing experimental results. It is hardly necessary to add that the philosophy underlying such texts is scientifically legitimate and that they will undoubtedly stay in the mainstream of combustion science for many years to come. Nevertheless, we are of the opinion that there is need for texts that treat combustion as a mathematical science and the present work is an attempt to meet that need in part.

In this monograph we describe, within a mathematical framework, certain basic areas of combustion science, including many topics rightly covered by introductory graduate courses in the subject. Our treatment eschews sterile rigor inappropriate for a subject in which the emphasis has been physical, but we are deeply concerned with maintaining clear links between the mathematical modeling and the analytical results; irrational approximation is carefully avoided. All but the most fastidious of readers will be satisfied that the mathematical conclusions are correct, except for slips of the pen.

Although the material covered inevitably reflects our special interests and personal perspectives, the entire discussion is connected by a singular perturbation procedure known as activation-energy asymptotics. The description of reacting systems characterized by Arrhenius kinetics can be simplified when the activation energy is large, corresponding to an extreme sensitivity to temperature. The notion is an old one; certainly the Russian combustion school was aware of it by the '40s. But its full power is only achieved within the formal framework of modern singular perturbation theory. Developments of this nature are quite recent. Although the earliest example is possibly a 1962 study of P. A. Blythe on the flow of a reacting gas through a Laval nozzle, it was not until the '70s, following a seminal paper by W. B. Bush and F. E. Fendell (on the laminar flame speed) and the call by F. A. Williams in the 1971 *Annual Review of Fluid Mechanics*, that extensive applications to combustion theory were undertaken.

Development has been increasingly rapid. A few years ago this monograph would not have been possible; but now there are more than enough results in a wide range of topics to justify the connected account attempted here. Our hope is that, by collecting many of the results together in a single volume with unified and self-contained presentation, other applied mathematicians will be encouraged to enter the field; for, although activation-energy asymptotics is no panacea, many exciting discoveries undoubtedly lie ahead.

Our discussion is concerned only with these modern developments; we have made no attempt to trace the history of the subject before the advent of activation-energy asymptotics. No disparagement is intended. On the contrary, this new tool has been used on problems that were largely identified, formulated, and discussed by the giants of the past; those who developed the material used in this monograph stood firmly on broad shoulders. Our failure to trace the earlier work stems from the conviction that it is better left to someone more scholarly than we are, someone who has been immersed in the subject longer than we have. (Similar remarks apply to our neglect of parallel problems in other areas, such as chemical reactor theory.)

Our treatment of the physical aspects of the subject requires some comment. The neophyte is well advised to read a traditional text in conjunction with ours, since phenomenology plays only a minor role in our presentation. The description of a physical phenomenon by clear-cut mathematical analysis is applied mathematics at its best and this ultimate justification of our efforts is not neglected in the discussion. But we are never in danger of letting the existing physical world be both judge and jury for the mathematical results. Where the results do not presently appear to have convincing physical counterpart, we say so; but no topic has been omitted for lack of such a counterpart. The value of rational deductions from physically plausible models is not always immediately apparent and applied mathematics is ultimately most fruitful when permitted a great deal of independence.

Reaction-diffusion equations arise copiously elsewhere, but activation-energy asymptotics has been largely restricted to combustion. In biochemical contexts, temperature effects (when present) are of a different character. For chemical reactors, problems involving large activation energy are far less common than for combustion. While significant results for chemical reactors have been obtained by activation-energy asymptotics, and more will be, it seems unlikely that the coverage will ever be as extensive as for combustion.

This monograph is not a review of laminar flames for large activation energy but an essay on the subject. The text only mentions works that helped us in writing a connected account of the topics chosen. To recognize other contributions and to give an idea of the wealth of results that has been obtained, we have compiled a list of further references. Substantial use of activation-energy asymptotics was a necessary qualification; in particular, treatments of thermal ignition in the style of Frank-Kamenetskii were excluded. Such a list is bound to be incomplete, especially since no serious attempt was made to search the literature. Any omissions are due to carelessness or ignorance, but not to malice. Only published work available to us by the end of 1981 is included; in particular, theses do not appear.

The notation has been kept as simple as possible. Rather than conscripting exotic symbols, we have made latin and greek letters do multiple duty; the meaning of a symbol should be clear from context. We have also been economical in the designation of and reference to chapters, sections, figures, and displays (so as not to litter the page with numbers): a decimal is used only for reference to another chapter. When part of a multiple display is intended, the letters *a*, *b*, . . . are added to indicate the first, second, . . . part. Finally, the notation used uniformly throughout the text for coefficients in asymptotic expansions is explained on page 19.

The task of writing this book has been lightened and made more pleasant by the enthusiastic encouragement we have received from so many friends and colleagues. We particularly want to thank and identify those who provided direct technical help, all of which was invaluable; they are

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The monograph was started six years ago during a sabbatical at the University of Queensland, arranged by A. F. Pillow. Solid progress was made during a joint leave at the Mathematics Research Center–Wisconsin in the Fall of 1977, arranged by B. Noble. The work was finished early this year, but we have tried to up-date the material as the printing process slowly ran its course.

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