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978-0-521-09188-6 - An Introduction to Riemannian Geometry and the Tensor Calculus

C. E. Weatherburn

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