Cambridge Tracts in Mathematics
and Mathematical Physics

No. 59

PROXIMITY SPACES

PROXIMITY SPACES

S. A. NAIMPALLY
Professor of Mathematics
Indian Institute of Technology
Kanpur

AND

B. D. WARRACK
Department of Mathematics
University of Alberta

CAMBRIDGE
AT THE UNIVERSITY PRESS
1970

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521079358

First published 1970
This digitally printed version 2008

A catalogue record for this publication is available from the British Library

Library of Congress Catalogue Card Number: 73–118858

ISBN 978-0-521-07935-8 hardback
ISBN 978-0-521-09183-1 paperback

CONTENTS

PREFACE

This tract aims at providing a compact introduction to the theory of proximity spaces and their generalizations. It is hoped that a study of the tract will better enable the reader to understand the current literature. In view of the fact that research material on proximity spaces is scattered and growing rapidly, the need for such a survey is apparent. The material herein is self-contained except for a basic knowledge of topological and uniform spaces, as can be found in standard texts such as the one by John L. Kelley; in fact, for the most part, we use Kelley's notation and terminology.

The tract begins with a brief history of the subject. The first two chapters give the fundamentals and the pace of development is rather slow. We have tried to motivate definitions and theorems with the help of metric and uniform spaces; a knowledge of the latter is, however, not necessary in understanding the proofs. The main result in these two chapters is the existence of the Smirnov compactification, which is proved using clusters. Taking advantage of hindsight, several proofs have been considerably simplified.

A reader not acquainted with uniform spaces will find it necessary to become familiar with such spaces before reading the third chapter. In this chapter, the interrelationships between proximity structures and uniform structures are considered and, since proximity spaces are intermediate between topological and uniform spaces, some of the most exciting results are to be found in this part of the tract. Various generalizations of uniform spaces find their way naturally into the theory presented here. The final chapter deals with several generalized forms of proximity structures, with one of them being studied in some detail. This chapter is rather sketchy and the interested reader is referred to the relevant literature for further information.

In order to minimize the number of discontinuities occurring in the main body of the text, all references from which material is selected as well as those where further details can be found are collected together in the Notes at the end of each chapter. We have attempted to provide a reasonably complete bibliography of the literature on proximity spaces; to this end we were greatly aided by D. Bushaw's *Bibliography on Uniform Topology* (Washington State University, November, 1965). At the end of each item in the bibliography is found a reference to Mathematical Reviews. Appended separately is a list of general references used in the tract. An index of notations and another of terms are also included.

With great pleasure we acknowledge our indebtedness to several colleagues. Dr K. M. Garg, Mr C. M. Pareek, Professor A. J. Ward and Professor K. Iséki assisted with advice during the initial stages. Comments by Professor C. T. C. Wall on the first draft of the manuscript were useful during revision. Several mathematicians kindly sent us their unpublished manuscripts; we are especially grateful to Dr C. J. Mozzochi, who also made several suggestions. Mathematical manuscripts are difficult to type and we admire the skill and patience of our typists: Miss June Talpash, Mrs Vivian Spak and Mrs Georgina Smith.

The first author would like to take this opportunity to express gratitude to his inspiring teachers: Professors D. S. Agashé, M. L. Chandratreya, D. P. Patravali, N. H. Phadke from India, and Professors J. G. Hocking and D. E. Sanderson from the U.S.A. This author was generously supported by operating grants from the National Research Council (Canada) and the Summer Research Institute of the Canadian Mathematical Congress (1967).

We thank the staff of the Cambridge University Press for their help and cooperation. Miss M. Gagrat helped in the difficult task of proof correction.

S. A. N.
B. D. W.

May 1969
The University of Alberta
Edmonton, Canada

INDEX OF NOTATIONS