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S. A. Naimpally and B. D. Warrack
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PREFACE

This tract aims at providing a compact introduction to the theory of proximity spaces and their generalizations. It is hoped that a study of the tract will better enable the reader to understand the current literature. In view of the fact that research material on proximity spaces is scattered and growing rapidly, the need for such a survey is apparent. The material herein is self-contained except for a basic knowledge of topological and uniform spaces, as can be found in standard texts such as the one by John L. Kelley; in fact, for the most part, we use Kelley's notation and terminology.

The tract begins with a brief history of the subject. The first two chapters give the fundamentals and the pace of development is rather slow. We have tried to motivate definitions and theorems with the help of metric and uniform spaces; a knowledge of the latter is, however, not necessary in understanding the proofs. The main result in these two chapters is the existence of the Smirnov compactification, which is proved using clusters. Taking advantage of hindsight, several proofs have been considerably simplified.

A reader not acquainted with uniform spaces will find it necessary to become familiar with such spaces before reading the third chapter. In this chapter, the interrelationships between proximity structures and uniform structures are considered and, since proximity spaces are intermediate between topological and uniform spaces, some of the most exciting results are to be found in this part of the tract. Various generalizations of uniform spaces find their way naturally into the theory presented here. The final chapter deals with several generalized forms of proximity structures, with one of them being studied in some detail. This chapter is rather sketchy and the interested reader is referred to the relevant literature for further information.

In order to minimize the number of discontinuities occurring in the main body of the text, all references from which material is selected as well as those where further details can be found are

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collected together in the Notes at the end of each chapter. We have attempted to provide a reasonably complete bibliography of the literature on proximity spaces; to this end we were greatly aided by D. Bushaw's *Bibliography on Uniform Topology* (Washington State University, November, 1965). At the end of each item in the bibliography is found a reference to Mathematical Reviews. Appended separately is a list of general references used in the tract. An index of notations and another of terms are also included.

With great pleasure we acknowledge our indebtedness to several colleagues. Dr K. M. Garg, Mr C. M. Pareek, Professor A. J. Ward and Professor K. Iséki assisted with advice during the initial stages. Comments by Professor C. T. C. Wall on the first draft of the manuscript were useful during revision. Several mathematicians kindly sent us their unpublished manuscripts; we are especially grateful to Dr C. J. Mozzochi, who also made several suggestions. Mathematical manuscripts are difficult to type and we admire the skill and patience of our typists: Miss June Talpash, Mrs Vivian Spak and Mrs Georgina Smith.

The first author would like to take this opportunity to express gratitude to his inspiring teachers: Professors D. S. Agashé, M. L. Chandratreya, D. P. Patravali, N. H. Phadke from India, and Professors J. G. Hocking and D. E. Sanderson from the U.S.A. This author was generously supported by operating grants from the National Research Council (Canada) and the Summer Research Institute of the Canadian Mathematical Congress (1967).

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May 1969
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INDEX OF NOTATIONS

		<i>page</i>
Iff	If and only if	
$A \subset B$	$x \in A$ implies $x \in B$	
δ	proximity relation	8
$\tau(\delta)$	topology induced by proximity δ	8
A^δ	$\{x: x\delta A\}$	11
$\text{Int}(A)$	interior of A	
\bar{A} or $\text{Cl}(A)$	closure of A	
$A \ll B$	B is a δ -neighbourhood of A	15
δ_Y	subspace proximity on Y	23
$\prod_{a \in I} X_a$	product of sets X_a , for $a \in I$,	23
σ_x	point cluster determined by x	28
\mathcal{L}	ultrafilter	27
σ	cluster	27
\mathcal{X}	set of all clusters in X	39
$\bar{\mathcal{A}}$	set of all clusters containing A	39
$w(\tau)$	topological weight	47
$ \mathcal{B} $	cardinal number of \mathcal{B}	
$w(\delta)$	proximity weight	48
N	set of natural numbers	
\mathcal{U}	uniformity	63
Δ	diagonal, i.e. $\{(x, x): x \in X\}$	63
$\tau(\mathcal{U})$	topology induced by a uniformity \mathcal{U}	64
$\delta(\mathcal{U})$	proximity induced by a uniformity \mathcal{U}	64
\mathcal{U}_Y	subspace uniformity on Y induced by \mathcal{U}	68
$\mathcal{M}(\sigma)$	see paragraph following (11.4)	68
$\Pi(\delta)$	proximity class of uniformities	71
$\mathcal{W} = \mathcal{W}(\delta)$	unique totally bounded member of $\Pi(\delta)$	72
\bar{U}^n	For $U \subset X \times X$, $\bar{U}^0 = U$ and $\bar{U}^{n+1} = \bar{U}^n \circ U$ for each $n \in N$	63
$\mathcal{U} \vee \mathcal{V}$	supremum of uniformities \mathcal{U} and \mathcal{V}	64
$\Lambda(\delta)$	proximity class of A-N uniformities	79
$\mathcal{U} \leq \mathcal{V}$	\mathcal{U} is less than or equal in height to \mathcal{V}	84
$H(\mathcal{U})$	height class of \mathcal{U}	84

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\mathcal{U}_w	$\mathcal{W}(\delta)$, where $\delta = \delta(\mathcal{U})$	page 85
$H(X)$	hyperspace of all closed subsets of X	87
\mathcal{U}	hyperspace uniformity induced on $H(X)$ by uniformity \mathcal{U} on X	87
$<$	topogenous order	98
Σ	generalized topological (or GT -) structure	98
δ_Σ	proximity associated with GT -structure Σ	99
\mathbf{n}	sequential proximity	100
C_I	first countable or first axiom of countability	
(F_n)	denotes a net or a sequence of functions	
(x)	constant sequence, each element of which is x	
$\{f_n\}$	range of (f_n)	
$(f_n: n \in D)$	net of functions f_n on a directed set D	94
α	Leader (LE-) or Lodato (LO-) proximity	105
β	Pervin or P-proximity	105
ξ	arbitrary generalized proximity	105
R_0	separation axiom ($x \in \bar{y}$ iff $y \in \bar{x}$)	106
\mathcal{Q}	quasi-uniformity	107
γ	separation or S-proximity	108
$U_{A,B}$	$X \times X - [(A \times B) \cup (B \times A)]$	113
$\Gamma(\delta)$	LO-proximity class of M-uniformities	113
A^c	complement of A	
B^A	the set of all functions from A to B	98