

CONTENTS

	PAGE
PREFACE	v
INTRODUCTION. HISTORICAL SUMMARY	1
CHAP.	
I. MÖBIUS TRANSFORMATION	
§ 5. Conformal representation in general	3
§§ 6–9. Möbius Transformation	4
§§ 10–12. Invariance of the cross-ratio	5
§§ 13–15. Pencils of circles	7
§§ 16–22. Bundles of circles	8
§§ 23–25. Inversion with respect to a circle	11
§§ 26–30. Geometry of Möbius Transformations	13
II. NON-EUCLIDEAN GEOMETRY	
§§ 31–34. Inversion with respect to the circles of a bundle	16
§ 35. Representation of a circular area on itself	17
§§ 36, 37. Non-Euclidean Geometry	18
§§ 38–41. Angle and distance	19
§ 42. The triangle theorem	21
§ 43. Non-Euclidean length of a curve	22
§ 44. Geodesic curvature	22
§§ 45–47. Non-Euclidean motions	23
§ 48. Parallel curves	25
III. ELEMENTARY TRANSFORMATIONS	
§§ 49–51. The exponential function	26
§§ 52, 53. Representation of a rectilinear strip on a circle	27
§ 54. Representation of a circular crescent	28
§§ 55–59. Representation of Riemann surfaces	29
§§ 60, 61. Representation of the exterior of an ellipse	31
§§ 62–66. Representation of an arbitrary simply-connected domain on a bounded domain	32
IV. SCHWARZ'S LEMMA	
§ 67. Schwarz's Theorem	39
§ 68. Theorem of uniqueness for the conformal representation of simply-connected domains	40
§ 69. Liouville's Theorem	40
§§ 70–73. Invariant enunciation of Schwarz's Lemma	41
§ 74. Functions with positive real parts	43
§ 75. Harnack's Theorem	44
§ 76. Functions with bounded real parts	45
§§ 77–79. Surfaces with algebraic and logarithmic branch-points	45

CHAP.		PAGE
IV.	§§ 80–82. Representation of simple domains	46
	§§ 83–85. Representation upon one another of domains containing circular areas	50
	§ 86. Problem	52
	§§ 87, 88. Extensions of Schwarz's Lemma	52
	§§ 89–93. Julia's Theorem	53
V. THE FUNDAMENTAL THEOREMS OF CONFORMAL REPRESENTATION		
	§ 94. Continuous convergence	58
	§§ 95, 96. Limiting oscillation	58
	§§ 97–99. Normal families of bounded functions	61
	§ 100. Existence of the solution in certain problems of the calculus of variations	62
	§§ 101–103. Normal families of regular analytic functions	63
	§ 104. Application to conformal representation	66
	§§ 105–118. The main theorem of conformal representation	66
	§ 119. Normal families composed of functions which transform simple domains into circles	73
	§§ 120–123. The kernel of a sequence of domains	74
	§ 124. Examples	77
	§§ 125–130. Simultaneous conformal transformation of domains lying each within another	77
VI. TRANSFORMATION OF THE FRONTIER		
	§§ 131–133. An inequality due to Lindelöf	81
	§§ 134, 135. Lemma 1, on representation of the frontier	82
	§ 136. Lemma 2	84
	§§ 137, 138. Transformation of one Jordan domain into another	85
	§§ 139, 140. Inversion with respect to an analytic curve	87
	§§ 141–145. The inversion principle	88
	§§ 146–151. Transformation of corners	91
	§§ 152, 153. Conformal transformation on the frontier	96
VII. TRANSFORMATION OF CLOSED SURFACES		
	§§ 154, 155. Blending of domains	98
	§ 156. Conformal transformation of a three-dimensional surface	99
	§§ 157–161. Conformal representation of a closed surface on a sphere	100

CONTENTS

ix

CHAP.	PAGE
VIII. THE GENERAL THEOREM OF UNIFORMISATION	
§§ 162, 163, 164. Abstract surfaces	103
§§ 165, 166. The universal covering surface	104
§ 167. Domains and their boundaries	105
§ 168. The Theorem of van der Waerden	106
§ 169. Riemann surfaces	108
§§ 170, 171. The Uniformisation Theorem	110
§ 172. Conformal representation of a torus	111
BIBLIOGRAPHICAL NOTES	113