87 Exponential diophantine equations
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Exponential diophantine equations
To our parents and our wives
Contents

Preface vii
Introduction 1
Notation 5
Preliminaries 9
A. Results from algebraic number theory
B. Estimates of linear forms in logarithms 29
C. Recurrence sequences 32
Diophantine equations and recurrence sequences 40
1. Purely exponential equations
2. Binary recurrence sequences with rational roots 56
3. Binary recurrence sequences 63
4. Recurrence sequences of order 2, 3 and 4 82
5. The Thue equation 99
6. The superelliptic equation 113
7. The Thue–Mahler equation 124
8. The generalised superelliptic equation 141
9. Perfect powers in binary recurrence sequences 150
10. Perfect powers at integral values of a polynomial 169
11. The Fermat equation 184
12. The Catalan equation and related equations 201
References 221
Index 239
Preface

After the appearance of Baker's fundamental papers 'Linear forms in the logarithms of algebraic numbers' in Mathematika in 1966–8, Baker, Coates and others obtained upper bounds for the magnitudes of integer solutions of some polynomial diophantine equations in two unknowns and their p-adic generalisations. The finiteness of the numbers of solutions of these equations had been proved by Thue, Siegel, Mahler and others much earlier. The publication of Baker's papers 'A sharpening of the bounds for linear forms in logarithms' in Acta Arithmetica in 1972–5, and van der Poorten's p-adic analogues of it, led to completely new results on exponential diophantine equations such as the work on the Catalan equation by Tijdeman and its p-adic analogue by van der Poorten. Since the numerous publications on exponential diophantine equations are scattered over journals and no thorough introduction is available, we have decided to write a tract on these results.

We were together at the University of Leiden in 1982–3 for one year. A first draft of the manuscript was written during this period. The subsequent work of finalising the manuscript was carried out by correspondence spread over a period of about two years. The stay of one of us (T.N.S.) at the University of Leiden was supported in part by the Netherlands Organisation for the Advancement of Pure Research (Z.W.O.).

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Preface

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R.T.