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A First Course in
MATHEMATICAL STATISTICS

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A First Course in
**MATHEMATICAL
STATISTICS**

by

C. E. WEATHERBURN

M.A., D.Sc., Hon. LL.D.

*Emeritus Professor of Mathematics in the University
of Western Australia*

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To

PROFESSOR R. A. FISHER
IN ADMIRATION OF HIS
CONTRIBUTIONS TO MATHEMATICAL STATISTICS
AND TO THE MEMORY OF
PROFESSOR KARL PEARSON
WHO PLAYED AN OUTSTANDING PART
IN THE EARLIER DEVELOPMENT OF THE SUBJECT

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P R E F A C E

The object of this work is to provide a mathematical text on the Theory of Statistics, adapted to the needs of the student with an average mathematical equipment, including an ordinary knowledge of the Integral Calculus. The subject treated in the following pages is best described not as Statistical Methods but as Statistical Mathematics, or *the mathematical foundations of the interpretation of statistical data*. The writer's aim is to explain the underlying principles, and to prove the formulae and the validity of the methods which are the common tools of statisticians. Numerous examples are given to illustrate the use of these formulae; but, in nearly all cases, heavy arithmetic is purposely avoided in the desire to focus the attention on the principles and proofs, rather than on the details of numerical calculation.

The treatment is based on a course of about sixty lectures on Statistical Mathematics, which the author has given annually in the University of Western Australia for several years. This course was undertaken at the request of the heads of some of the science departments, who desired for their students a more mathematical treatment of the subject than those usually provided by courses on Statistical Methods. The class has included graduates and undergraduates whose researches and studies were in Agriculture, Biology, Economics, Psychology, Physics and Chemistry. On account of such a diversity of interest the lectures were designed to provide a mathematical basis, suitable for work in any of the above subjects. No technical knowledge of any particular subject was assumed.

The first five chapters deal with the properties of distributions in general, and of some standard distributions in particular. It is desirable that the student become familiar with these, before being confronted with the theory of sampling, in which he is required to consider two or more distributions simultaneously. However, the reader who wishes to make an earlier start with sampling theory may take Chapters VI and VII (as far as §60) immediately after Chapter III, since these are independent of the

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theory of Correlation. The theory of Partial and Multiple Correlations has been left for the final chapter, in order not to delay the study of sampling theory and tests of significance. But those who wish to study this subject earlier may read this chapter immediately after Chapter v, or even after Chapter iv. This order, however, is not recommended for the beginner.

A feature of the book is the use of the properties of Beta and Gamma variates in proving the sampling distributions of the statistics, which are the basis of the common tests of significance. Consequently a special chapter (VIII) is devoted to the properties of these variates and their distributions. The treatment is simple, and does not assume any previous acquaintance with the Beta and Gamma functions. The author believes that the use of these variates brings both simplicity and cohesion to the theory. The student is strongly urged to master the theorems of Chapter VIII before proceeding to tests of significance. In the preparation of this chapter, and the following one, much help was derived from the study of a recent paper by D. T. Sawkins.*

The considerations, which determined the presentation of the subject of Probability in Chapter II, are the mathematical attainments of the students for whom the book is intended, and their requirements in studying the remaining chapters. The approach decided on is substantially that of the classical theory. After the proof of Bernoulli's theorem, the relation between the *a priori* definition of probability and the statistical (or empirical) definition is considered; and the measure of probability by a relative frequency in each case is emphasized. If the book had been intended primarily for mathematical specialists, a different presentation of the theory would have been given. But it is futile to expect the average research worker to appreciate an exposition like Kolmogoroff's† or Cramér's,‡ based on the theory of completely additive

* 'Elementary presentation of the frequency distributions of certain statistical populations associated with the normal population.' *Journ. and Proc. Roy. Soc. N.S.W.* vol. 74, pp. 209-39. By D. T. Sawkins, Reader in Statistics at Sydney University.

† *Grundbegriffe der Wahrscheinlichkeitsrechnung*, Berlin, 1933.

‡ *Random Variables and Probability Distributions*, University Press, Cambridge, 1937.

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set functions. The elementary properties of the moment generating function and the cumulative function are also given in Chapter II, and are used throughout the book. These functions are introduced, not as essential concepts, but as useful instruments which lead to simpler proofs of various theorems. What has been written about them should convince the reader that they deserve his careful attention.

I wish to express my appreciation of the care bestowed on this book by the staff of the Cambridge University Press, and my pleasure in the excellence of the printing. My thanks are due to Professor R. A. Fisher and Messrs Oliver and Boyd for permission to print Tables 3, 4, 5 and 7, which are drawn from fuller tables in Fisher's *Statistical Methods for Research Workers*. I am also indebted to Professor G. W. Snedecor and the Iowa Collegiate Press for permission to reproduce Table 6, which is extracted from a more complete table in Snedecor's *Statistical Methods*. Lastly I wish to thank Mr D. T. Sawkins for help received in correspondence concerning statistical theory, and Mr Frank Gamblen for assistance in reading the final proof.

C.E.W.

PERTH, W.A.
April, 1946

NOTE ON THE SECOND EDITION

The call for reprinting has given an opportunity to correct a number of small errors and misprints throughout the book, and to add a new reference here and there. Paragraph 91 on page 195 is new to this edition.

1949

C.E.W.