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978-0-521-09156-5 - The Methods of Plane Projective Geometry Based on the Use of General Homogeneous Coordinates

E. A. Maxwell

Index

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INDEX

- Algebraic**
 correspondence, 23–5
 parameter, 22
- Angle**
 between two circles, 193
 bisectors, 188
 corresponding, 187
 defined by cross-ratio (Laguerre), 185–6
 in segment of circle, 187
 right, 187
- Asymptotes, 195**
- Axis**
 of central conic, 196
 of parabola, 199
- Base of range, 51**
- Brianchon, 95, 174 et seq.**
- Cartesian coordinates, 1, 2; homogeneous, 180, 182**
- Centre**
 circle, 192
 conic, 195; locus for conics of tangential pencil, 201
- Centroid, 207**
- Chasles, 112–14**
- Circle, 192–5**
 angle between two circles, 193
 angle theorems, 187
 centre, 192
 centres of similitude, 205
 circular points at infinity, 178, 182, 203, 205 et seq.
 circumcentre, 206
 coaxial, 194
 diameter, 192
 escribed, 207–8
 form $P^2 + \lambda\Omega = 0$, 203
 inscribed, 207–8
 intersection of two circles, 178–82
 limiting points, 194
 nine-points, 201, 208
 orthogonal circles, 193
 radical axis, 194, 206, 209
 radius, 192
 tangent, 192–3, 194; tangents common to two circles, 205
- Circular points at infinity, 178, 182**
 by equation in line-coordinates, 203
 intersection of line and conic, 205 et seq.
- Complex numbers, xix**
- Confocal. See Conic**
- Conic**
 analytical method using S, S_1, S_{11} , etc., 149–52
 associated envelope and locus, 80, 86–7
 asymptotes, 195
 axis, 196, 199
 centre, 195; centres collinear for tangential pencil, 201
 Chasles's theorem, 112–14
 chord, 60, 100; joining points in involution, 65; joining points in (1, 1) correspondence, 115–18
 circumscribed to triangle of reference, 93
 confocal central conics, 198; confocal parabolas, 200
 confocal system, $\Sigma + \lambda\Omega = 0$, 204
 conjugacy, 76
 conjugate lines, 79; conjugate points, 76; conjugate diameters, 195–6
 constructions (*see* Ruler constructions)
 contact of envelope, 78, 80
 correspondence on conic, 111–21
 cross-ratio on conic, 112–15
 defined by two related pencils of lines, 112
 degenerate, 72–3, 81–7, 112, 144–5; condition for degeneracy, 82; pencils of degenerate conics, 141–3
 determined by five points, 72–3
 diameters, 195
 directrix, 196, 199
 double contact, 116, 143, 146–8, 163–5
 duality, 64, 77–81, 160
 eccentricity, 197
 eleven-points conic, 135–7
 ellipse, 195
 envelope, 77–80; envelope-pole and envelope-polar, 79

Cambridge University Press

978-0-521-09156-5 - The Methods of Plane Projective Geometry Based on the Use of General Homogeneous Coordinates

E. A. Maxwell

Index

[More information](#)

228

INDEX

Conic (*continued*)

equation in form $y^2 = zx$, 99–101
 equation in general form, 71, etc.
 equation of conic about triangle of reference, 93–4
 equation of conic in triangle of reference, 95
 equation of conic, triangle of reference self-polar, 96–8
 equations of two conics, 132; in line-coordinates, 133
 foci, 196, 198, 204, 205
 four-point contact, 117, 143, 146, 149
 harmonic conic, 157–8; harmonic envelope, 155–7
 harmonic separation on a conic, 118
 harmonically circumscribed conics, 167–8
 harmonically inscribed conics, 168
 hyperbola, 195
 inscribed in triangle of reference, 95
 intersections of two conics in four points, 143, 179. *See also* double, three-point, four-point contact
 involution cut on line by pencil of conics, 134
 involution on conic, 65, 116, 118, 119–20
 involution determined by tangential pencil, 134–5
 Joachimstal's equation, 73; dual form, 78
 line-coordinates: equation, 79–80, 93, 95, 97, 100; of chord, 62; of tangent, 62
 line touching conic, condition, 76–7
 locus, 77, 79
 metrical properties (*see* Asymptotes, Axes, Centre, etc.)
 normal, 195
 notation S , S_1 , S_{11} , etc., 71
 one-one correspondence on conic, 111–21
 pairs of conics without common self-polar triangle, 148–9
 parabola, 195, 198–200
 parametric form ($\theta^2 \theta, 1$), 67, 99–101
 parametric treatment, 60 et seq.
 pencils of conics, 130, 134, 141 et seq., 200–1
 pole and polar, 75–6, 79, 81, 84–6; polar as chord of contact, 76; ruler construction, 129; with respect

to degenerate conic, 84–7; with respect to conics of a pencil, 135–7, 151

range of conics, 132

reciprocation, 160–5

rectangular hyperbola, 195, 201

self-polar (self-conjugate) triangle, 96, 97; common self-polar triangle of conics of pencil, 133, 135, 147; conic for which two triangles in given conic are self-polar, 168

tangent, 62, 74, 80; coincidence of two tangents from point of conic, 64, 75; intersection of two tangents, 63, 66

tangential equation of point-pair in which line cuts conic, 88

tangential pencil, 132, 134, 137, 201

three-point contact, 143, 146, 149

touch, conics which, 143, 144–5

Conjugate

diameters, 195–6

harmonic, 50, 53, 55, 129

points with respect to a conic, 76

Contact

double, 116, 143, 146–8, 163–5

four-point, 117, 143, 146, 149

of conic-envelope, 78, 80

three-point, 143, 146, 149

Coordinates, 1–4

cartesian, 1

cartesian homogeneous, 1, 180, 182

homogeneous coordinates explained, 3

line-coordinates, 12. *See also* Conic transformation, 8–9, 14, 67

Correspondence

algebraic, 23–5. *See also* One-one algebraic correspondence

Corresponding angle, 187

Cross-ratio

convenient expression, 45–6

defined, 43

equianharmonic, 46

harmonic, 46, 48, 49

invariance for (1, 1) correspondence, 44

involution, 48, 54

of (1, 1) correspondence, 48; of four elements, 45; of pencil and range, 52; on conic (Chasles), 112

values, 43, 46; values not all distinct, 46

Curve, 60

Cambridge University Press

978-0-521-09156-5 - The Methods of Plane Projective Geometry Based on the Use of General Homogeneous Coordinates

E. A. Maxwell

Index

[More information](#)

INDEX

229

- Desargues, 9
- Determinants
for conic treated parametrically, 60;
for general equation of conic,
71; properties assumed, including
minors, xvii; vanishing of deter-
minant for conics, 68, 81–7
- Directrix, 196, 199
- Double elements of involution, 35
- Duality, 11–12, 64, 77–81, 160
- Eccentricity, 197
- Elation, 30
- Envelope, 77–9
- Equation (*see* Conic, Line, Point, etc.)
- Equianharmonic, 46
- Euclidean, 3, 4, 178 et seq.
- Feuerbach, 208
- Foci
central conic, 196; parabola, 198
involution, 35
- Harmonic (for conic properties, *see*
under Conic)
conjugate, 50, 53, 55; ruler construc-
tion, 129
cross-ratio, 46, 48, 49
pencil, 53
properties of quadrangle and quadri-
lateral, 128–9
property of involution, 48
range, 53
separation, 48–9, 50, 53; on conic, 118
tests for harmonic separation, 50, 53,
54–5
triangle, 126
- Hessian
points of triad, 33–4, 121
- Hexagon, 174 et seq.
- Homogeneous
coordinates explained, 3
equation, 2, 64
parameters, 21, 180
- Homography, 24
- Homologous elements, 24
- Imaginary points, 180–2, 190–2
- Infinite values, xviii
- Infinity
circular points at, 178–82, 203, 205,
and throughout Chaps. xi, xii
line at, 182, 183
points at, 178, 182, 183, 190–2
- Involution
condition for three pairs in, 37
conic properties (*see* Conic)
cross-ratio of, 48
cross-ratio test for self-corresponding
element, 54
definition, 34
determination by two pairs, 35–6
double elements, 35
equation, 35, 36
foci, 35
illustrations, 38
mates, 35
self-corresponding elements, 35, 36,
54–5
unique pair common to two, 37–8
- Joachimstal's equation, 73; use, 74, 75,
82, 83
dual form, 78
- Laguerre, 186
- Length
defined by cross-ratio, 183–4
on different lines, 189
- Line
at infinity, 182, 183
coordinates of, 12
definition, 4
equation of, 4–5
line-pair. *See* Conic, degenerate
pencil of lines, 51–4
properties, 5–7
through point, 13
- Mates, in involution, 35
- Metrical (*see* Angle, Length, Parallel
lines, etc. for details)
interpretation, Chap. xi
properties of figures, Chap. xii
- Middle point, 185
of diagonals of quadrilateral, 201
- Nine-points circle, 201, 208
- Normal, 195
- One-one algebraic correspondence,
Chap. ii
algebraic treatment, 25 et seq.
conic, 111–21
convenient choice of parameter, 31,
32–3
cross-ratio of, 48

Cambridge University Press

978-0-521-09156-5 - The Methods of Plane Projective Geometry Based on the Use of General Homogeneous Coordinates

E. A. Maxwell

Index

[More information](#)

230

INDEX

- One-one algebraic correspondence
(*continued*)
defined, 23–4
determination by three pairs, 30
direction of, 24
elation, 30
equation in simple form, 28, 29, 32–3
exceptional case $ad = bc$, 26–7
pairs common to two, 31
parabolic, 30, 49
parameters, 21, 27, 31, 32–3
self-corresponding elements, 25, 27
self-corresponding elements not distinct, 29, 33, 49
- Orthocentre, 201, 207
- Orthogonal, 193
- Osculate, 146
- Pappus, 11
- Parabolic correspondence, 30, 49
- Parallel lines, 181
interpretation, 183
transversal, 187
- Parallelogram, 185
- Parameters (*see* Conic, Homogeneous, One-one algebraic correspondence, etc.)
- Pascal, 94, 121, 174 et seq.
- Pencil
of conics, 130–1; Chap. ix
of lines, 51–4; harmonic, 53
tangential, 132, 134, 137, 201
- Perspective, 9–10
- Plane, Euclidean, complex projective, 3, 178, 190–2
illustrations, 202–3
- Point
coordinates of, 3
equation of, 12–13, 14
point-pair (*see* Conic, degenerate)
- Polar line with respect to triangle, 10
- Pole and polar, 75, 79. *See also* Conic
- Poristic systems
of quadrilaterals, 171–3
of triangles, 167–70
- Projectively related elements, 24
- Projectivity, 24
- Quadrangle and quadrilateral
cyclic, 187
definitions, 125
diagonals, 125; diagonal triangle, 125, 126, 130, 131
equations, 126–7
harmonic properties, 128
harmonic triangle, 126
inscribed in conic, 175
opposite sides or vertices conjugate for conic, 127–8
poristic system, 171–3
- Radical axis, 194, 206, 209
- Range
harmonic, 53
of conics, 132
of points, 51
- Ratios, use of, xviii, 2, 21
- Reciprocation, 160–5, 169
- Rectangular hyperbola, 195
- Reality, xix, 190–2
- Related, 24
- Right angle, 187
- Ruler constructions
harmonic conjugate, 129
pole and polar, 129
tangents to conic, 176
- Self-corresponding elements (*see* One-one algebraic correspondence)
- Self-polar (self-conjugate) triangle (*see* Conic)
- Similitude, centres of, 205
- Tangent, 62, 74, 80
ruler construction, 176
- Tangential equation
definition, 81
of point-pair in which line cuts conic, 88
- Todd, Dr J. A., 26
- Transformation, 8–9, 14, 67
for conics, 67
- Triad, 33, 121
- Triangle, defined, 7
of reference, 7
poristic systems of triangles, 167–70
two triangles in one conic, about another, 169
- Uniqueness of parameter, 22
- Unit
line, 9, 10
of length, 184
point, 8, 10
- Vertex of pencil, 51
of triangle, 7