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978-0-521-09155-8 - Elementary Matrices: And Some Applications to Dynamics and
Differential Equations

R. A. Frazer, W. J. Duncan and A. R. Collar

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ADDENDA ET CORRIGENDA

Additional Definitions.

The *trace* of a square matrix is the sum of the elements in the principal diagonal. It is equal to the sum of the latent roots.

Characteristic vector or *proper vector* is equivalent to “modal column” or “modal row”, though less explicit. The term “eigenvector” is sometimes used but is to be strongly deprecated.

p. 33. *Special types of square matrix.* Add

“Matrix is *unitary* when $u^{-1} = \bar{u}'$.”

A real unitary matrix is orthogonal but an orthogonal matrix need not be real.

All the latent roots of a unitary matrix have unit modulus.

Matrix is *persymmetric* when the value of u_{ij} depends only on $(i + j)$.”

p. 110. Insert dagger † against footnote.