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978-0-521-09155-8 - Elementary Matrices: And Some Applications to Dynamics and Differential Equations

R. A. Frazer, W. J. Duncan and A. R. Collar

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