CAMBRIDGE TRACTS IN MATHEMATICS
AND MATHEMATICAL PHYSICS

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63. *Contiguity of probability measures: some applications in statistics*

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Contiguity of probability measures: some applications in statistics

CAMBRIDGE
AT THE UNIVERSITY PRESS
1972

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521083546

First published 1972
This digitally printed version 2008

A catalogue record for this publication is available from the British Library

Library of Congress Catalogue Card Number: 71–171682

ISBN 978-0-521-08354-6 hardback
ISBN 978-0-521-09095-7 paperback

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Acknowledgement

Grateful thanks are due to the University of California Press for permission to use the material in Chapter 6, which is a version of a talk given by the author at the Sixth Berkeley Symposium on Mathematical Statistics and Probability. A paper based on this address has appeared in *Sixth Symposium of Mathematical Statistics and Probability*, edited by Lucien LeCam and published by the University of California Press.

Preface

'Although this may seem a paradox, all science is dominated by the idea of approximation.'

Bertrand Russell

This monograph represents a modest attempt on my part to introduce the concept of contiguity, elaborate on the mathematical theory behind it, and also indicate some of its statistical applications. It lays no claim in containing an exhaustive discussion of results pertaining to contiguity. In fact, there are already new results available which, however, could not have been included in this book. It is simply the result of an attempt to make the concept of contiguity and some of its statistical applications more familiar to several kinds of research workers. These include Theoretical Statisticians, Probabilists, Mathematicians whose primary interest lies in measure theory or approximation theory, and perhaps to practitioners of Statistics as well. It is my belief that contiguity deserves more attention than it has received. I hope that this monograph will be a step in that direction, anticipating the appearance of a more comprehensive treatise on the subject.

The concept of contiguity was introduced by Professor Lucien LeCam as a criterion of nearness of sequences of probability measures. In addition to its purely mathematical interest, contiguity is a powerful and very useful tool in Statistics, when one is concerned with asymptotic theory, leading to elegant derivations of the asymptotic properties of tests and estimates under less restrictive assumptions than usual. Of course, this presupposes that one subscribes to the usefulness of large sample theory results. Frequently, however, this is the only alternative open to the Statistician in an unfriendly real world.

Preface

The concept of contiguity is introduced in Chapter 1 and its relationship to some other modes of nearness of sequences of probability measures is investigated. Some general results which are based on contiguity and which form the backbone of the later chapters are also derived here. In Chapter 2, the statistical model to be employed throughout this monograph is introduced. This is that of a Markov process satisfying certain mild regularity conditions. Of course, the important independent identically distributed case is included in the Markovian model as a special case. Among the assumptions made here the most important one is that of differentiability in quadratic mean of a certain random function. This replaces, in effect, certain of the classical assumptions made in the literature. We refer to those assumptions which deal with the existence of pointwise derivatives up to the third order, and their boundedness by integrable random variables, of essentially the same random function referred to earlier. Under the regularity conditions imposed on the model some general results regarding the asymptotic expansion, in the probability sense, and the asymptotic normality of the log-likelihood function are derived under both a fixed and a moving sequence of parameter points. Incidentally, these derivations attest to the powerfulness and the elegance of the contiguity approach. The essence of Chapter 3 rests on an exponential approximation to the likelihood function with a view to taking advantage of the optimal inference procedures available for the exponential family in a wide variety of problems. The subsequent Chapters 4 and 5 exhibit some statistical applications. Some testing hypotheses problems for real-valued parameters are discussed in Chapter 4. In Chapter 5, the problem of asymptotic efficiency of sequences of estimates is discussed to a certain extent both from the classical point of view as well as along the lines suggested by Wolfowitz. Section 3 of Chapter 4, may be studied directly after Chapter 2. The results included in Sections 5 and 6, however, presuppose knowledge of the material in Chapter 3. In Chapter 6, a multi-parameter testing hypothesis problem is discussed and, finally, in the Appendix various results used in the body of the monograph are gathered together. Most of them are also proved.

Preface

The first serious thought and attempt to organize the results of this monograph in their present form was made when I was visiting the Mathematics Institute of Aarhus University, Denmark, during the spring semester of 1969. The financial support of the Institute is gratefully acknowledged here. Some financial support was also provided by the University of Wisconsin Research Committee and the National Science Foundation when many of the results included here were written up as Technical Reports. This is also gratefully acknowledged.

Some of the material of this monograph has been discussed in seminars both in Aarhus University and the University of Wisconsin in Madison. Comments of the participants in these seminars helped in modifying the original proofs of some theorems. In connection with this, I wish to thank O. Barndorff-Nielsen, R. A. Johnson, E. Spjøtvoll and G. K. Bhattacharyya. Many thanks are due to B. Lind for his invaluable contribution in helping clarify and organize various parts of this monograph. Thanks also go to A. Philippou and A. Soms for their many helpful suggestions and constructive comments. W. Davis is also to be thanked for a number of useful comments as well as all those who attended my lectures and contributed their comments. Professor J. F. C. Kingman read the first draft of this monograph on behalf of the Cambridge University Press. His thoughtful comments have contributed to improve the presentation of the material in the monograph. Also the Cambridge University Press itself has been very helpful throughout the editorial and publication process. I extend my sincere thanks to both. Last, but not least, I am grateful to Professor Lucien LeCam who introduced me to the subject of contiguity.

Madison, Wisconsin
February 1972

GEORGE G. ROUSSAS