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978-0-521-09094-0 - Entropy, Compactness and the Approximation of Operators

Brend Carl and Irmtraud Stephani

Frontmatter

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This book deals with a branch of functional analysis that has developed over the last decade. Entropy quantities are connected with the 'degree of compactness' of compact or precompact spaces, and so are appropriate tools for investigating linear and compact operators between Banach spaces. The main intention of this Tract is to study the relations between compactness and other analytical properties, e.g. approximability and eigenvalue sequences, of such operators. The authors present many new and generalized results, some of which have not appeared in the literature before. In the final chapter, the authors demonstrate that, to a certain extent, the geometry of Banach spaces can also be developed on the basis of operator theory.

All mathematicians working in functional analysis and operator theory will welcome this work as a reference or for advanced graduate courses.

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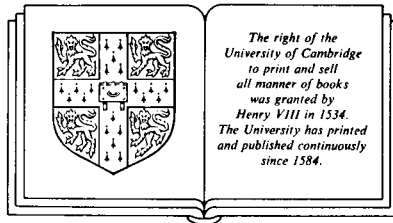
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BERND CARL

IRMTRAUD STEPHANI

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Preface

This book deals with a branch of modern functional analysis which has arisen only in the last 10 years, although it has its origin in a 1932 paper by Pontrjagin and Schnirelman.

In general there is quite a big difference between the level of recent research and the level of lectures as they are given to students. The question arises if this is in the nature of the subject, or if it is mainly a problem of producing an appropriate representation of the subject. Concerning 'Entropy, compactness and the approximation of operators', we came to the opinion that it should be possible to represent the subject at a level which makes reference only to the results of an introductory course on functional analysis. We have tried to write the book in the corresponding style and have listed in the introduction the concepts necessary for an understanding of the book. A few facts beyond the standard elementary knowledge of functional analysis are used without proof. However, a reader who is only interested in the fundamental relations between entropy quantities, approximation quantities, and eigenvalues can leave out the more difficult passages. By reading only sections 1.1, 1.2, 1.3, 1.4 of chapter 1, section 2.1 of chapter 2, section 3.1 of chapter 3, and section 4.2 of chapter 4, he or she will get an impression of the main ideas of the book and will be able to follow the applications of the general results in chapter 5. A course along these lines could be considered complementary to classical functional analysis, in particular to Riesz theory, or to classical approximation theory.

A book is never the work of the author alone, not even if this term stands for a group of two or more people. We had the help of our colleagues Dr Stefan Geiss, Dr Albrecht Hess, Dr Thomas Kühn, and Dr Doris Planer and want to thank them for reading the manuscript and making critical comments. In particular we are obliged to Dr Albrecht Hess who identified himself with the book project and the intentions of the authors. He discovered errors that we had overlooked and improved some earlier drafts. Miss Heike Gierschner typed the text excellently. Many thanks also to her!

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Last but not least we wish to express our gratitude to Cambridge University Press. Professor Garling as an adviser to the Press recommended the book for publication in ‘The Cambridge Tracts’ series and suggested the final title. Publishing director David Tranah was a most reliable contact over a period of three years, meeting our expectations in all respects. We do hope that the reader will be as happy with the final product as we are with all the people who supported us.

Oldenburg
Jena, April 1990

Bernd Carl
Irmtraud Stephani