The theory of classical dynamics

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PREFACE

The purpose of this book is to describe in detail the theory known as classical dynamics. This is a theory which is very well known and has a large number of important practical applications. It is regarded as one of the most basic scientific theories, with many other theories being direct developments or extensions of it. However in spite of its acknowledged importance, it is not always as well understood as it ought to be.

In the English speaking world at least, the emphasis in the teaching of classical dynamics is largely on its relevance to idealised applications. Thus most traditional mechanics textbooks are example orientated, and students are required to work through large numbers of artificial exercises. Other textbooks are available which emphasise the mathematical techniques that are appropriate in the applications of the subject. These approaches are of course of great importance. An understanding of this subject can only be achieved by working through numerous examples in which the theory is applied to practical problems. However, I feel that a discussion of the fundamental concepts of the theory and its basic structure has largely been neglected.

Classical dynamics is far more than an efficient tool for the solution of physical and engineering problems. It is a fascinating scientific theory in its own right. Its basic concepts of space, time and motion have fascinated some of the greatest intellects over many centuries. My aim in writing this book is to encourage students to think and to gain pleasure from their intellectual studies, as well as provide them with an essential tool for the solution of many practical problems.

The theory of classical dynamics is one of the most basic of all scientific theories. Almost all modern physical theories include concepts carried over from it. So my aim is to describe the theory in detail, paying particular attention to the basic concepts that are involved, as well as to the way in which the theory is applied.

Writing in the latter part of the twentieth century, it is possible to use not only recent studies in the philosophy of science, but also the additional insights into the subject of dynamics that have come from the development of the modern theories of relativity and quantum mechanics. It is in fact one of my concerns in this book to indicate the limitations of the classical theory, and hence also to specify those areas in which it may confidently be applied.

I have omitted a discussion of the historical development of the subject. There already exist many excellent volumes covering this material. I include here only an analysis of the mature theory in the present form in which it is understood and applied.

Although the main purpose of this book is to expound the theory of classical dynamics, I do not believe that this can adequately be done without considering a number of examples. Accordingly, I have included a number of separate sections of worked examples, which are followed by exercises that are left to the reader to complete. To obtain a deep understanding of the theory it is necessary both to consider real situations and to work through more idealised exercises of the type included here.

With the combination here of the theory with examples and exercises, I hope that this book will be found appropriate as a recommended or supplementary text for university courses in both mathematics and physics departments. A certain amount of mathematical notation has necessarily been used throughout the book, and this has accordingly been set at the level of a first-year undergraduate course in mathematics or physics.

INTRODUCTION

The classical theory of dynamics is that scientific theory which was developed through the seventeenth, eighteenth and nineteenth centuries to describe the motion of physical bodies. The theory was originally developed in two distinct parts, one dealing with terrestrial bodies such as a projectile or a pendulum, and the other dealing with celestial bodies, in particular the planets. These two apparently distinct subjects were first brought together by Isaac Newton whose book, known as the *Principia*, is rightly acknowledged as the first complete formulation of the theory that is now referred to as *classical dynamics*.

Since those early days the theory has been thoroughly developed and extended so that it can now be applied to a very wide range of physical situations. Some of the early concepts have been clarified, and others have been added. But, although the theory has now been formulated in many different ways, it is still essentially the same as that originally proposed by Newton. The most significant advances have in fact been those associated with the development of new mathematical techniques, which have subsequently enabled the theory to be applied to situations which previously had proved too difficult to analyse.

Now, as is well known, a revolution occurred in scientific thinking in the first part of the present century. Classical theories were disproved, and exciting new theories were put forward. The theory of relativity was suggested in order to explain the results obtained when the speed of light was measured. Such experimental results could not be accounted for using classical theories, but the new theory involved replacing the concepts of space and time that had been accepted for centuries. Then a new theory of gravitation was put forward that involved the concept of a curved space–time. This was a great shock to a generation that had been schooled on Euclid. Yet the new theory was shown to be in agreement with observations, while the classical theory was not. Also, a whole new theory of atomic structure with subatomic particles was developed. This was the theory of quantum mechanics, which has led to a much deeper understanding of the nature and structure of matter. Old concepts were modified or abandoned and a whole new science emerged.

These new theories, however, are extremely complicated mathematically and in order to apply them to the motion of reasonably sized bodies, it is necessary to use many approximations. In most situations of this type, the mathematical approximation of the new theories is identical to that which is also obtained from the much simpler theory of classical dynamics. In such situations it is therefore appropriate simply to continue to apply the classical theory. In practice it can thus be seen that the new theories of relativity and quantum mechanics, far from replacing classical theory, actually support it. In addition, since they also clarify the limiting situations in which the various theories become significantly different, they clarify the range of physical situations in which the classical limit can be applied. They therefore enable the classical theory to be applied with greater confidence within this range.

It is the purpose of this book to carefully explain the theory of classical dynamics. This is to be done by describing in detail the concepts that are required in the theory and the way in which it can logically be applied. The two main reasons for this are the obvious ones. Firstly, since the classical theory continues to be applied in a vast number of different situations both in science and engineering, it is necessary that the theory should be understood and applied appropriately and correctly. Secondly, most of the concepts that were introduced first in classical dynamics such as mass, energy and momentum, are taken over and used in many other theories. For this and other reasons, the theory of classical dynamics is regarded as one of the most fundamental of all scientific theories. This being so, it is very important that its methodology and basic concepts should be properly understood.

In order to understand any theory, it is imperative that it should be stated as clearly and accurately as possible, and the most appropriate way to state a scientific theory is in axiomatic form. Once the basic axioms of a theory are stated, its implications can be deduced using the standard methods of deductive logic and mathematical reasoning.

Such an approach displays both the essential character of the theory and the way in which it is to be applied.

It is important though to realise that the axioms of a scientific theory are not 'self-evident truths'. They are, rather, simply the assumptions or conjectures upon which the theory is based. They are the foundational statements which are taken on trust and used without question in the development and application of the theory. However, since they are essentially conjectural, they may be questioned, and it is sometimes instructive to consider replacing them by alternatives which would give rise to different theories making different sets of predictions.

In order to emphasise their character, the basic axioms of classical dynamics are stated in this book as a series of *assumptions* and *definitions*. From these the basic results of classical dynamics can be deduced. The most important of these are stated formally here as a series of *propositions*. Since the theory can be summarised in terms of these statements, they are separately listed for convenience in the appendix.

Now the purpose of any scientific theory is to describe what actually happens in the real world. So the more accurately a theory describes real events, the closer it may be assumed to be to the truth, and the greater the confidence that can be placed in its foundational assumptions. Because of its great success, considerable confidence has been placed in the foundational axioms of classical mechanics. These relate particularly to the assumed nature of space and time, to the concept of force, and to the particle and many-particle representations of real bodies. However, with the greater success of the modern theories of relativity and quantum mechanics, which each have some basic axiom which differs significantly from those of classical theory, some of the foundational axioms of classical dynamics have been shown to be false, and certain alternatives are now considered to be closer to the truth.

From the above discussion it can be seen that it is most convenient to adopt an axiomatic approach in order both to describe the theory of classical dynamics, and also to indicate the points of departure which lead to the theories of relativity and quantum mechanics. However, the main aim here is to expound the theory as clearly as possible rather than to present an axiomatic formulation of it. Thus no attempt has been made to demonstrate the usual requirements of axiomatic systems that they be consistent, independent and complete. In fact in at least one case the basic assumptions are not independent. However, it is hoped that the essential character of the theory is clearly demonstrated.

The theory of classical dynamics has been developed over the centuries so that it is now applicable to a very wide range of physical phenomena. It has in addition been directly extended to cover such other subjects as classical electrodynamics and magneto fluid mechanics. These and many other theories can also be formulated in terms of the generalised theories of analytical dynamics. The subject of this book, however, is only the classical theory of dynamics. The extensions of the theory are not considered. The theories of Lagrangian and Hamiltonian dynamics are therefore described here only in terms of their origins in classical mechanics, and no attempt is made to develop them to their full generality. It is, however, necessary to treat these theories in some detail both because of their use in applications, and because of the additional insight that they give into certain aspects of the classical theory.