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The Newtonian method

As with other scientific theories, theories of dynamics aim to contribute to an understanding and explanation of phenomena that occur in the real world. In the theories of dynamics, it is the general subject of motion that comes under investigation. The aim is to describe how objects move, and to suggest physical reasons as to why they move. In particular they should provide methods for analysing or predicting the motion of specific bodies, and also possibly suggest techniques for controlling the motion of some objects.

The theory known as classical or Newtonian dynamics, the subject of this book, is one such theory. Before describing it in detail, however, it is convenient to describe in general terms the way in which the theory is used. This is the purpose of this first chapter.

1.1. The technique of mathematical modelling

The basic method by which any theory of dynamics is applied can be described in terms of three distinct phases. The first phase consists essentially of constructing a simplified model. This is an idealised imaginary representation of some physical situation in the real world. In phase two this theoretical representation is analysed mathematically and its consequences are deduced on the basis of some assumed theory. Finally, in phase three, the theoretical results of phase two are interpreted and compared with observations of the real physical situation. This whole process can of course be repeated many times using different initial representations or different basic theories. However, it is usually only repeated until the results obtained at phase three are considered to be sufficiently accurate for the purpose that was originally envisaged. This general method of approach is that known as mathematical modelling, and is the basic method of all so-called applied mathematics. However, since the basic theory being applied also regulates the whole modelling process, it is appropriate to describe this process here in a little more detail in terms of the particular way in which it is used in conjunction with the theory of classical dynamics.

As an initial illustration of this method, we may consider how the motion of a planet can be described and its position at any time predicted. In the construction of a mathematical model it may be appropriate to begin with to regard the planet as a point mass, to assume that it is only acted on by a gravitational force directed towards the sun, and to regard the sun as a fixed point. Such a model can then be analysed mathematically and, using the Newtonian theory of gravitation and Newton's laws of motion, it is found to predict that the planet will follow an elliptic orbit about the sun. There are, however, a whole family of possible ellipses, and so a series of observations are required, first to determine the parameters which specify the particular ellipse, and then to check that the planet does in fact continue to follow such an ellipse, at least approximately. Once this is confirmed, it is possible to predict its position at any later time within a certain estimated error bound. This whole process can then be repeated using more realistic models. For example, the planet and sun can be represented as bodies of finite size, the sun may be considered to move, and the effects of other planets or other perturbing forces can also be introduced. It is also possible to change the basic theory and to attempt to follow the same procedure using the general theory of relativity. Using this approach it is ultimately found to be possible to determine the position of any planet at any time with a remarkable degree of accuracy.

With this illustration in mind, it is now appropriate to consider the separate phases of this approach in greater detail, paying particular attention to the way in which it is applied to the analysis of the motion of real bodies.

In phase one the physical situation being considered is represented by an idealised mathematical model. Such a representation is purely theoretical. It transfers attention from the real physical world to some imaginary construct in the mathematical domain where it may be analysed in terms of mathematical equations. It is therefore necessary that the representations of the various parts of the physical system should be stated in terms that can be expressed mathematically. It is usually found, however, that if a fully detailed representation is considered the mathematics becomes unwieldy and the model cannot be analysed. It is therefore essential that the model adopted is a much simplified and idealised representation of its real counterpart. The art of mathematical modelling lies in choosing a model that is sufficiently simple to be analysed easily, but yet is sufficiently detailed to accurately reflect the behaviour of the real physical system. In practice, however, a number of models are usually considered. Very simple models may first be used to describe the general qualitative behaviour of a system, or to provide initial approximations. Then progressively more detailed models may be considered until the degree of mathematical sophistication required becomes out of proportion to the improvement of results achieved.

Even at this initial stage it is necessary to decide which theory is going to be applied, since the choice of theory usually also affects the choice of possible representations. Here it is assumed that the theory to be applied is that of classical dynamics, and so the representation is stated in terms of mass distributions, forces, frames of reference, initial velocities and perhaps internal or external constraints. These are all regulated by the theory and will be discussed in later chapters.

The choice of classical mechanics as the appropriate basic theory immediately specifies the type of mathematical models to be considered, and the range of features of the real situation that are ignored. For example, a physical body is usually represented in this theory solely in terms of its assumed mass distribution. Other properties, such as its colour, are assumed to be totally irrelevant. Even properties such as its temperature, or chemical composition, are ignored unless they are thought to affect its mechanical properties. It can thus be seen that this approach leads to a theoretical model that is necessarily only a restricted or partial representation of its real counterpart.

To further simplify the model it is always necessary to theoretically isolate the systems being considered from all other aspects of the real-world situation that are not considered to be relevant to its motion. This aspect is similarly regulated by the theory. For example, as far as classical mechanics is concerned, the theory indicates immediately the significant forces which may be assumed to act on the system. Other less significant forces need not be included in the model. Thus, in discussing the motion of a planet, the position of its moons, the other planets and the stars may be ignored, at least initially.

Finally, having isolated a particular system, it is still necessary to simplify or idealise its representation in order to analyse it mathematically. For this reason it is often appropriate to consider such fictions as the point masses, light inelastic strings, smooth planes or rigid bodies. The planet, for example, may initially be represented as a single particle. Together these various stages of idealisation essentially consist of sets of simplifying assumptions which in applications enable particular physical systems to be represented simply in precise and manageable mathematical terms.

Once such an idealised model has been constructed, it is then analysed mathematically. This is the second phase of the modelling process. In it the basic theory is applied explicitly. In this case it is the theory of classical dynamics that is to be applied, and this is usually stated in terms of equations of motion. Although there are alternative formulations, the equations of motion of classical dynamics can usually be expressed as, or at least result in, sets of ordinary differential equations. The various terms of these equations and the parameters they contain are all determined by the theory when applied to the particular model. In fact it is often convenient to initially describe the model in terms of certain characteristics or expressions that can be substituted directly into the equations of motion. This again illustrates the point that the basic theory affects the way in which a physical system is represented in the model.

At this point it may be assumed that the mathematical model has been described in terms of sets of ordinary differential equations. Apart from sometimes having to evaluate expressions or characteristics to substitute into these equations, the main effort in this phase is to attempt to integrate the equations and to investigate the mathematical properties of their solutions.

Ever since the theory was first put forward, much of the research associated with it has been directed towards the development of mathematical techniques that are particularly appropriate to the type of equations that occur in applications of the theory. In spite of this, however, it still usually happens in practice that complete analytic solutions of the equations of motion cannot be obtained. Attention is therefore directed towards the possible occurrence of first integrals, or constants of the motion, such as the energy integral or integrals of momentum. Particular techniques have been developed in order to derive such integrals.

In addition, various approximation techniques may also be applied. These may be of two types. They may simply involve the numerical integration of the equations of motion as they stand, for particular initial conditions. The errors that arise in such calculations are purely of a mathematical nature and can easily be estimated. The alternative approach is to relate the approximation technique more directly to the physical situation. This essentially involves a further simplification of the model in which a parameter which is known to be small is temporarily assumed to be zero. If an analytic solution of this simplified model can be obtained, the parameter may then be reintroduced and a better approximate solution obtained using perturbation techniques.

In the final phase of the modelling process, the mathematical results obtained in the model are reinterpreted in terms of the real physical system under consideration. In some ways this can be seen as the opposite process to the initial construction of the model. Mathematical constructs are again related to physical objects in the real world, and the behaviour of the model can be compared with observed physical processes.

The results obtained from a mathematical model are referred to as predictions. Essentially they predict what the physical system would do if it behaved exactly like the model. If it is possible to observe the motion of the physical system, then these observations may be compared with the predictions, and if these are roughly in agreement the model is said to be satisfactory.

In practice, however, the comparison of predictions and observations is a little more complicated, since the mathematical model, and the predictions derived from it, usually contain a number of unspecified parameters. These have to be determined before specific predictions can be made. This can frequently be achieved simply by observing the physical system for a period of time and choosing the parameters to match the model to the system. Often, however, this process is more complicated, and, since this forms an essential step in the testing of scientific theories, it is appropriate to consider it here in greater detail.

1.2. The testing of models and theories

As has been stated above, the results or predictions obtained from mathematical models are usually described in terms of a number of arbitrary parameters. These arise in a variety of ways. To start with, certain quantities or parameters are frequently introduced in the theory itself. Examples of such are the mass or moment of inertia of each body, or Newton's constant of gravitation. These are not always specified explicitly in the initial construction of the model, although some may be calculated directly from it. In addition, the integration of the equations of motion may also result in the introduction of further arbitrary constants.

It is in fact usually regarded as advantageous for models to contain a number of unspecified parameters, since this enables them to represent a range of physical situations. Models are often deliberately constructed in this way, although they do not aim at total generality. That is the aim of the basic theory. Scientific theories are developed to cover as wide a range of physical processes as possible. It is the application of such theories to particular types of situations within that range that yields theoretical models. Thus the purpose of the model is to represent a certain class of situation within the broader range that is covered by the theory.

The behaviour of general models like this which contain arbitrary parameters can be compared with the corresponding range of physical situations. However, to obtain more than a general qualitative agreement special cases have to be considered. In order to do this it is necessary to specify the particular values of the parameters which correspond to each particular case being considered. The various parameters, however, are determined in a number of different ways.

Some of the parameters can frequently be determined directly from observation of the physical situation. For example, the initial positions and velocities of certain parts of the system can often be observed and their values used to specify some of the parameters. In other cases it is necessary to observe the motion of the system over a period of time before some parameters are specified. In such cases the comparison of the model with observations is used to clarify or specify the model rather than test it. Finally, there are also cases in which certain parameters have to be estimated from separate experiments based on other, or related, theories and their associated models. Sometimes it is necessary to perform such experiments. For example, the mass of a small object can sometimes be estimated separately by taking it aside and weighing it. Alternatively, the generally accepted results of others may be used. For example, the mass of the planets, or the value of Newton's constant of gravitation, are now considered to be well known.

It should, however, be emphasised that the particular values of the parameters that are inserted in the model are only estimates of the corresponding physical quantities. It is only possible to determine them up to a certain specified degree of accuracy. Thus the mathematical models based on the theory of classical dynamics are only capable of providing an approximate representation of a particular system. However, since the accuracy of each parameter can be determined, it is possible to predict the behaviour of the system to within certain specified error bounds.

Such predictions can now be compared with the actual observed behaviour of the physical system. However, the observations cannot be made with perfect accuracy either. Observed quantities can only be estimated to within certain specified error bounds.

Observations of a particular system can now be compared with the predictions obtained from a particular model. If these are found to be in agreement within the estimated errors, then the model is clearly satisfactory for this particular case. Once the model has been corroborated for one such case, it may then be tested for other particular cases. Finally, if the model has been found to be satisfactory for a number of situations of a particular type having parameters within a certain specified range, it may then be used with a certain amount of confidence for other situations of the same type within that range. It is possible to use the model to predict what would happen for various values of the parameters in the acceptable range. Such predictions may then be assumed to be correct, even though the physical situation may not have actually been observed for every particular case.

It is of course also possible to use an established model to describe situations in which the values of some parameters lie outside the range for which the model has been tested. The results obtained in such cases should, however, only be accepted with a certain reservation until the extrapolation has been tested.

At this point it is convenient to consider what should be done if the predictions of a model do not agree with observations of the physical system it is supposed to represent.

In such cases it is first necessary to check the accuracy of the observations, the accuracy of the estimates for any parameters including initial conditions that are required in the model, and also the accuracy to which any calculations are performed. The possible errors arising in each of these cases can be calculated, and we must now consider the possible situation in which predictions and observations still do not agree, even after allowing for these possible errors.

At this point it may also be assumed that the disagreement is not due to any trivial mistake, but that repeated observations of a particular case consistently indicate a significant difference between predictions and observations. It is therefore only possible to conclude that the model is unsatisfactory. The problem now is to find the unsatisfactory aspects of the model, and to attempt to correct them.

Now, when a particular model is unsatisfactory in the sense described above, it is almost always found that this is due to some oversimplification in its original construction. Sometimes the initial assumptions are wrong, or at least are not sufficiently accurate. In such cases more accurate initial assumptions should be tried. In other cases it is often found that some significant factor has been ignored. In all such cases it is necessary to return to phase one of the modelling process and to consider a more accurate model.

Sometimes it is convenient to retain the initial model as an approximate representation. It frequently happens that, in order to obtain a model that is perfectly satisfactory, terms have to be included which make the mathematical analysis extremely complex. In such situations simpler approximate models are usually used. Even though they are known to be technically unsatisfactory, they may still give answers that are sufficiently accurate for some purposes. Approximate models of this type are particularly useful once it is known, at least in principle, how to correct them. In such cases it is possible to estimate the effect of the terms that have been omitted. Thus the errors which arise in the model as a result of the simplification can be explicitly determined.

Although it only rarely occurs in practice, it is still necessary to consider the remaining possibility that the observations of a system may not agree with the predictions obtained from a model, even after the model has been corrected and made as accurate as possible. In such a case it can only be concluded that it is the basic theory that is at fault, and an alternative theory should be considered.

It is this final possibility that enables scientific theories to be put to the test. However, before considering any particular theory to be disproved or falsified, it is necessary always to check thoroughly that the disagreement does not arise from the oversimplification of the model. The particular way in which this approach can be used to test the theory of classical dynamics is described in more detail in section 3.5. A further example in terms of the Newtonian theory of gravitation is also described in section 5.5.

Now, part of the scientific method is that every theory, and the models based upon it, should be tested as thoroughly as possible. This is done in principle as described above. However, this does not mean

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that endless experiments have to be performed to check each model for more and more values of its associated parameters. Instead, the theory is tested in two ways. The first method is to attempt to reduce the error bounds in both the model and the observations. This quest for greater and greater accuracy is made possible by technological advances in both observational instruments and experimental apparatus. The second method, which is also associated with it, is the quest to extend the range over which the models are tested.

For a theory to be generally accepted it should be capable of providing models which, in principle at least, are entirely satisfactory. In addition, it should be applicable to a wide range of physical situations. Both of these features are demonstrated by the theory of classical dynamics. To start with it is applicable to an exceptionally wide range of physical problems, both terrestrial and celestial, and this range can easily be extended to many situations that are outside the usual subject area of mechanics. In addition, within this range, it is capable of describing what happens in the real world to at least the accuracy that can be achieved by the most advanced instruments.

However in some extreme situations the theory of classical dynamics is found to fail. This occurs when considering systems in which the velocity of some components becomes a significant fraction of the speed of light, or when the size of the bodies considered are so small that they must be considered as atomic or subatomic particles. In these situations new theories have been established.

These new theories of relativity and quantum mechanics provide a different approach to the whole modelling process. However, when they are applied to situations in the range in which the classical theory has been so successful, they become so complicated that the mathematical analysis of the models usually becomes impossible. In this situation further simplifications have to be made, and it is usually found that the resulting models constructed in this way are identical to those which would have been obtained using the simpler classical theory. The reason for this is that the new theories claim to be deeper theories which are closer approximations to reality. It is therefore necessary that all the tests which previously have been considered to corroborate the classical theory, should also corroborate the new theories. So the new theories are required to approximate to the classical theory in the appropriate range, and should only deviate from it where it is found to be unsatisfactory. Thus it can be seen that the classical theory may continue to be used, and that the new theories also clarify the range of situations in which it can be applied with confidence.

1.3. The primitive base of classical dynamics

Having briefly described the approach of mathematical modelling, it is appropriate to consider in a little more detail how the model is related to the actual physical situation in the real world. On the one hand we have physical objects that can be seen and handled, and on the other we have mathematical symbols and equations.

In fact there is no unique way in which these two distinct types of object can be related. They exist in totally different worlds. Yet in order to apply the scientific method some correspondence must be assumed. This can in fact be achieved using a certain number of primitive concepts. These form the foundational base upon which the theory is built.

The foundational concepts of classical dynamics are those of space, time, a physical body and force. These concepts are not defined exhaustively in the theory, as are, for example, angular momentum or kinetic energy. The question: 'What is kinetic energy?' can be answered satsifactorily in purely mechanical terms, once some basic aspects of the theory are understood. On the other hand the question: 'What is time?' or 'What is a physical body?' immediately involve deep philosophical problems, and cannot even be considered without stepping outside the narrow subject area of classical dynamics.

It can be seen that these foundational concepts involve some assumed representation of certain aspects of the real world, that can be initially taken on trust and used to develop theories. They are undefined but meaningful concepts which enable aspects of the real world to be analysed scientifically.

The foundational concepts can themselves be the subject of a deeper level of scientific or metaphysical theories. However, they are not concepts that are taken from elsewhere and simply inserted as the basis of a theory. Rather, they are taken as vague and ill-defined concepts and given a precise interpretation. The philosophical problems are then temporarily forgotten and they are used in the theory in terms of exact mathematical models. Finally, the successes and failures of the theory in its many applications provides a commentary on the appropriateness of the assumptions concerning these foundational concepts. In this way it can be seen that the theory of classical