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978-0-521-09064-3 - Miniquaternion Geometry: An Introduction to the Study of Projective Planes

T. G. Room and P. B. Kirkpatrick

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