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T. G. Room

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A BACKGROUND
(NATURAL, SYNTHETIC AND ALGEBRAIC)
TO GEOMETRY

BY

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PREFACE

The classical format of a school text-book of Euclid's geometry, in which symbolic language is arranged in a fixed pattern of statements, has much to recommend it; even the final triumphant Q.E.D. is not pedagogically supererogatory. The use of symbols and pattern makes it possible to state compactly and precisely every step in the construction of a figure and the proof of a theorem. In this book the format has been adapted to a development of geometry in which 'collinearity' is the fundamental relation, while 'congruence' is an end-product, rather than, as in 'Euclid', an almost self-evident property.

Through the book there run three interwoven strands of development. The central strand might be described as 'making congruence respectable', and this was in fact the aim of the courses of lectures, first given twenty years ago, out of which the book has grown. Over the years the other two strands have been woven in. Along one of them is evolved a sequence of geometric constructions and theorems, based on combinatorial axioms for collinear subsets of a given set of points, which correspond to the operations of an algebraic field and their properties. The other strand follows some of the consequences of prescribing that the geometry shall consist of only finitely-many points, an area of mathematics still open for exploration at a relatively elementary level. Also, as a prelude to the geometry proper, there has been added some discussion in non-arithmetical terms of the ideas of 'order' and 'sense'.

In writing a book which grows, as this has done, out of courses of lectures given over many years, the author receives suggestions from many of his colleagues and from students. It is impossible to acknowledge these in detail, but the author wishes to express his deep sense of gratitude to all those colleagues and students in the University of Sydney who have contributed ideas to this book. There is one member of the Department of Mathematics to whom the author is especially indebted: Philip Kirkpatrick. His careful reading and criticism of both manuscript and proofs has resulted in the correction of many inaccuracies of statement as well as errors of typing and printing.

Because of the use made in the book of an unfamiliar symbolism and format, the printing has presented many difficulties; the sub-editorial staff of the Cambridge University Press has spared no pains in editing the manuscript so that the printers can print what the

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author intends. To them and to all members of the staff of the Press who have given help and guidance during the process of publication, the author wishes to offer his sincere thanks.

T. G. R.

Sydney, Australia
April 1967

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