Martingales and stochastic integrals

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Preface

Martingale theory is one of the most powerful tools of the modern probabilist. Its intuitive appeal and intrinsic simplicity combine with an impressive array of stability properties which enables us to construct and analyse many concrete examples within an abstract mathematical framework. This makes martingales particularly attractive to the student with a good background in pure mathematics wishing to find a convenient route into modern probability theory. The range of applications is enhanced by the construction of stochastic integrals and a martingale calculus.

This text has grown out of graduate lecture courses given at the University of Hull to students with a strong background in analysis but with little previous exposure to stochastic processes. It represents an attempt to make the 'general theory of processes' and its application to the construction of stochastic integrals accessible to such readers. As may be expected, the material is drawn largely from the work of Meyer and Dellacherie, but the influence of such authors as Elliott, Kussmaul, Neveu and Kallianpur will also be evident. I have not attempted to give credit for particular results: most of the material covered can now be described as standard, and I make no claims of originality. The Appendix by Chris Barnett and Ivan Wilde contains recent work on non-commutative integrals, some of which is presented here for the first time.

In general I have tried to follow the simplest, thus not always the shortest, route to the principal results, often pausing for motivation through familiar concepts. In emphasizing the use of functional analysis I have included a short description of the required results in Chapter 1, where a detailed proof of the Dunford–Pettis weak compactness criterion is also provided. This emphasis on analytic techniques aids the understanding of the analogies between the 'commutative and non-commutative' theories exploited in the

Appendix. A particular example is the treatment, following Neveu, of conditional expectations via orthogonal projections.

The brief treatment in Chapter 0 of Brownian motion and the Poisson process is intended to highlight the role of these processes as the traditional examples which lend substance to the abstract theory. Thus the discussion is very incomplete and largely intended to motivate later results.

The discussion of the principal features of discrete-parameter martingales in Chapter 2 is traditional, with somewhat more emphasis on the convergence theorems than is usual. The section on vector-valued martingales points towards applications in the geometry of Banach spaces. The final section, on optimal stopping, covers some quite recent results. Chapter 3 relies heavily on the exposition of the continuous-parameter theory given in [19], the standard treatise for all this and much besides. My choice of topics is guided by the principal application of the theory, namely stochastic integrals. The supplement on capacitability follows [18] in an attempt to give simple proofs of the fundamental 'theoremes de section', which are so often omitted in other texts.

The development of stochastic integrals for semimartingales in Chapter 4 follows [66] quite closely. Attention is also drawn to other approaches, e.g. those in [49] and [62] as well as [17]. A first course such as this does not allow detailed discussion of applications, and only the merest indications are given. Nonetheless I hope that this introduction will equip the reader to master current research literature in the many applications of this fast-expanding field.

Acknowledgments

David Edwards introduced me to martingales, Robert Elliott and Alfred Kussmaul have stimulated my interest in them over the years. Several generations of students suffered my attempts to interest them as well, and have helped my understanding of the subject. Particular thanks go to Nigel Cutland, who read the whole manuscript and made many valuable suggestions and improvements. Thanks are also due to Wilfrid Kendall for many improvements to Chapter 2, and to Ben Garling for a copy of his Cambridge lecture notes which helped to shape section 2.7. I must take responsibility for all remaining misconceptions.

I am indebted to Jennie Wilson and Alan Fleming who typed most of the manuscript, and to David Tranah of Cambridge University Press for suggesting the project and for his patient guidance during its execution.

It should be said that this book would have been completed earlier and with many fewer distractions had British universities been spared the pain of enforced 'contraction' in the wake of ill-conceived Government spending

cuts. Mathematics in Britain has indeed suffered far more grievous losses than this, but it remains galling for mathematicians to be forced into a defence of their subject in terms of 'market forces', when there are manifestly so many better things for them to do.

My family has been neglected unreasonably through my pre-occupations while preparing this book. Their support and affection has helped me to see it through to the end.

Hull, June 1983