

CONTENTS

<i>Preface</i>	<i>page ix</i>
1 Theory of sets	
1.1 Sets	1
1.2 Mappings	3
1.3 Cardinal numbers	5
1.4 Operations on subsets	9
1.5 Classes of subsets	14
1.6 Axiom of choice	19
2 Point set topology	
2.1 Metric space	23
2.2 Completeness and compactness	29
2.3 Functions	35
2.4 Cartesian products	38
2.5 Further types of subset	41
2.6 Normed linear space	44
2.7 Cantor set	49
3 Set functions	
3.1 Types of set function	51
3.2 Hahn–Jordan decompositions	61
3.3 Additive set functions on a ring	65
3.4 Length, area and volume of elementary figures	69
4 Construction and properties of measures	
4.1 Extension theorem; Lebesgue measure	74
4.2 Complete measures	81
4.3 Approximation theorems	84
4.4* Geometrical properties of Lebesgue measure	88
4.5 Lebesgue–Stieltjes measure	95
5 Definitions and properties of the integral	
5.1 What is an integral?	100
5.2 Simple functions; measurable functions	101
5.3 Definition of the integral	110
5.4 Properties of the integral	115
5.5 Lebesgue integral; Lebesgue–Stieltjes integral	124
5.6* Conditions for integrability	127

vi	CONTENTS	
6	Related spaces and measures	
6.1	Classes of subsets in a product space	<i>page</i> 134
6.2	Product measures	138
6.3	Fubini's theorem	143
6.4	Radon–Nikodym theorem	148
6.5	Mappings of measure spaces	153
6.6*	Measure in function space	157
6.7	Applications	162
7	The space of measurable functions	
7.1	Point-wise convergence	166
7.2	Convergence in measure	171
7.3	Convergence in p th mean	174
7.4	Inequalities	183
7.5*	Measure preserving transformations from a space to itself	187
8	Linear functionals	
8.1	Dependence of \mathcal{L}_2 on the underlying $(\Omega, \mathcal{F}, \mu)$	194
8.2	Orthogonal systems of functions	199
8.3	Riesz–Fischer theorem	202
8.4*	Space of linear functionals	209
8.5*	The space conjugate to \mathcal{L}_p	215
8.6*	Mean ergodic theorem	219
9	Structure of measures in special spaces	
9.1	Differentiating a monotone function	224
9.2	Differentiating the indefinite integral	230
9.3	Point-wise differentiation of measures	236
9.4*	The Daniell integral	241
9.5*	Representation of linear functionals	250
9.6*	Haar measure	254
10	What is probability?	
10.1	Probability statements	261
10.2	The algebra of events	265
10.3	Probability as measure	268
10.4	Conditional probability	272
10.5	Independent trials	277

CONTENTS		vii
11	Random variables	
11.1	Random variables as measurable functions	<i>page</i> 284
11.2	Expectations	286
11.3	Distributions of random variables	290
11.4	Types of distribution function	292
11.5	Independent random variables	295
11.6	Discrete distributions	301
11.7	Continuous distributions	306
11.8	Convergence of random variables	311
12	Characteristic functions	
12.1	The space of distribution functions	314
12.2	Characteristic functions	322
12.3	The inversion and continuity theorems	326
12.4	Generating functions	332
13	Independence	
13.1	Sequences of independent trials	335
13.2	The Borel–Cantelli lemmas and the zero-one law	337
13.3	Sums of independent random variables	340
13.4	The central limit theorem	348
13.5	The law of the iterated logarithm	351
14	Finite collections of random variables	
14.1	Joint distributions	356
14.2	Conditioning with respect to a random variable	359
14.3*	Conditioning with respect to a σ -field	363
14.4	Moments	365
14.5	The multinomial distribution	368
14.6	The multinormal distribution	370
15	Stochastic processes	
15.1	Renewal processes	376
15.2	The general theory of stochastic processes	380
15.3	Gaussian processes	384
15.4	Stationary processes	391
	<i>Index of notation</i>	395
	<i>General index</i>	397