# INTRODUCTION TO MEASURE AND PROBABILITY

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CAMBRIDGE UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521058889



First published 1966
Reprinted (with corrections) 1973
This digitally printed version 2008

*A catalogue record for this publication is available from the British Library*

*Library of Congress Catalogue Card Number: 66–12308*

ISBN 978-0-521-05888-9 hardback
ISBN 978-0-521-09032-2 paperback

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# PREFACE

We decided to write this book largely as a result of experience in teaching at the Instructional Conference on Probability held at Durham in 1963 under the auspices of the London Mathematical Society. It seemed that a proper treatment of probability theory required an adequate background in the theory of finite measures in general spaces. The first part of the book attempts to set out this material in a form which not only provides an introduction for the intending specialist in measure theory, but also meets the needs of students of probability.

The theory of measure and integration is presented in the first instance for general spaces, though at each stage Lebesgue measure and the Lebesgue integral are considered as important examples, and their detailed properties are obtained. An introduction to functional analysis is given in Chapters 7 and 8; this contains not only the material (such as the various notions of convergence) which is relevant to probability theory, but also covers the basic theory of $L_2$-spaces important, for instance, in modern physics.

The second part of the book is an account of the fundamental theoretical ideas which underlie the applications of probability in statistics and elsewhere. The treatment leans heavily on the machinery developed in the first half of the book, and indeed some of the most important results are merely restatements of standard theorems of measure theory. No attempt has been made to give a detailed description of the applications of the theory, and in particular there is no account of the problem of statistical inference, but it is hoped that this part of the book may provide the framework for a rigorous mathematical study of these subjects.

The book is designed primarily for final year Honours mathematicians at British universities, but might also serve as a text for graduate courses in measure theory, or in probability theory. We have included a large number of examples; these are not an optional extra, but form an essential part of the development, and include many facts which might have been stated as theorems. We hope that the book will also be found useful by those statisticians, and other applied mathematicians, who find a need to acquire a basic working knowledge of the mathematical foundations for the techniques which they use.

The book is largely self-contained. The first two chapters contain the essential parts of set theory and point set topology; these could well be omitted by a reader already familiar with these subjects. Chapters 3 and 4 develop the theory of measure by the usual process of extension from 'simple sets' to those of a larger class, and the properties of Lebesgue measure are obtained. The integral is defined in Chapter 5, again by extending its definition stage by stage, using monotone sequences, Chapter 6 includes a discussion of product measures and a definition of measure in function space. Convergence in function space is considered in Chapter 7, and Chapter 8 includes a treatment of complete orthonormal sets in Hilbert space. Chapter 9 deals with special spaces; differentiation theory for real functions of real variable is developed and related to Lebesgue measure theory, and the Haar measure on a locally compact group is defined.

Chapter 10 introduces the idea of probability, and Chapter 11 the concepts of a random variable and its distribution. In Chapter 12 characteristic functions are defined and their properties established; these are then used in Chapter 13 where the classical results on sums of independent random variables are presented. Chapter 14 deals with joint distributions of several random variables. Finally, in Chapter 15, the idea of a stochastic process and a few examples of such processes are discussed.

Starred sections contain more advanced material and can be omitted at a first reading.

It will be clear to any reader familiar with the standard treatises that this book owes much to what has gone before. We make no attempt to establish precedence and make detailed acknowledgements; nor do we claim any particular originality for our treatment; but the form of presentation owes a great deal to our experience in undergraduate teaching—at Cornell University, the University of London, and the University of Cambridge—and we readily acknowledge the stimulus received from these sources. We are also grateful to Mr B. Fishel and Professor G. E. H. Reuter, who among others made helpful criticisms of an early draft, and to Professor D. G. Kendall for his encouragement in the early stages of this project.

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# PREFACE TO SECOND IMPRESSION

We have taken the opportunity of correcting the misprints and errors in the first edition but we have resisted the temptation to carry out a major revision of the text. We are grateful to the many friends and readers who pointed out to us errors and obscurities which we have tried to correct.

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