

Cambridge University Press

978-0-521-09030-8 - Differential Analysis: Differentiation, Differential Equations and  
Differential Inequalities

T. M. Flett

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