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978-0-521-09030-8 - Differential Analysis: Differentiation, Differential Equations and
Differential Inequalities

T. M. Flett

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PREFACE

On 13 February 1976 Professor T. M. Flett died at the early age of 52. At that time he had almost completed the manuscript of the present book. In order that so much effort should not be lost, I undertook the task of finishing the work. My guiding principle has been that the book is still Professor Flett's: although I have no doubt that he would have made alterations in arriving at his own final version, I am sure that the reader would wish to hear his voice, even imperfectly, rather than mine. The text he left has been altered only where there were clear indications that he himself had intended to do this or on the rare occasions when errors had crept in. A few parts of the book which he clearly proposed to include did not exist even in manuscript. The reader will wish to know that I am solely responsible for §2.13, for the historical note on differentials (§4.7) and for the notes on chapters 3 and 4. In connection with the last two I probably owe apologies to many mathematicians. I do not have Professor Flett's encyclopaedic knowledge of the literature in this field and my attribution of theorems to their originators is not as detailed as his corresponding work for the earlier chapters; I fear that some names which should have appeared will have been omitted.

Professor Flett would have wished to thank Mr P. Baxendale who made valuable comments on a draft of Chapter 1. I myself am deeply indebted to Dr F. Smithies whose meticulous and painstaking examination of the whole manuscript saved me from many errors. Both Professor Flett and I owe much to Mrs E. Bennett who made our jobs easier by producing excellent typescripts of many drafts of the work.

J. S. Pym