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Contents

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Preface

I have been interested in analytic groups and algebraic structures on them for some time – ever since an anonymous reviewer of a grant proposal suggested I take a look at some papers of Hochschild and Mostow in connection with the work I proposed. I found the papers fascinating, and useful as I began work in the area, but the foundations of the subject seemed to me (as an autodidact Lie theorist) irrevocably recondite. Then, in the summer of 1978, two things happened: while preparing for a talk for a conference in Copenhagen, I found elementary arguments for the basic existential facts of the theory (these appear here in Chapter 3) and, more important, I met Alex Lubotzby at the conference, who pointed out the connection between the Hochschild–Mostow theory and the Grothendieck theory of Tannakian categories (this appears here in Chapter 2). These things meant that the subject was both easier and of wider application than I had previously imagined, and so inspired this volume.

The book was written while I enjoyed a sabbatical leave from The University of Oklahoma and was Visiting Professor of Mathematics, University of Virginia (Fall 1979), Visiting Professor of Mathematics, Bar-Ilan University (Winter 1980), and Visiting Professor of Mathematics, University of California, Berkeley (Spring 1980). To those institutions and my colleagues there, I am grateful. I especially want to acknowledge the encouragement I have gotten over the years from Gerhard Hochschild.

I am grateful to Trish Abolins for her able and efficient typing of the manuscript, to Hyman Bass for suggesting that the book appear as a Cambridge Tract, and to my family.

Chickasha, Oklahoma — A.R.M.
Thanksgiving, 1980

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Notation and conventions

$\mathbb{C}$ denotes the complex field.
Z denotes the integers.
All tensor products are over $\mathbb{C}$.
All vector spaces are over $\mathbb{C}$, and most are finite-dimensional.
If V is such a space, then:
V^* is the $\mathbb{C}$-linear dual of V;
$\mathrm{End}(V)$ is the space of $\mathbb{C}$-linear endomorphisms of V;
$\mathrm{GL}(V)$ is the group of $\mathbb{C}$-linear automorphisms of V;
$\mathrm{gl}(V)$ is $\mathrm{End}(V)$ as a $\mathbb{C}$-Lie algebra with bracket $[A, B] = AB - BA$;
$V^{\otimes n} = V \otimes V \otimes \cdots \otimes V$ (n times);
$\langle x_i \mid i \in S \rangle$ is the subspace of V spanned by the subset $\{x_i \mid i \in S\}$.
$\mathrm{GL}_n\mathbb{C}$ is the group of invertible $n \times n$ complex matrices.
$\mathrm{GL}_1\mathbb{C}$ is also denoted $\mathbb{C}^*$.
The diagonal $n \times n$ matrix with diagonal entries $a_1, \ldots, a_n$ is denoted $\mathrm{diag}(a_1, \ldots, a_n)$.

An analytic group is a connected complex Lie group.
The Lie algebra of the analytic group is denoted $\mathrm{Lie}(G)$, which is sometimes abbreviated to $L(G)$.
$X(G)$ is the group of analytic group homomorphisms from G to $\mathrm{GL}_1\mathbb{C}$ (the characters of G).
$X^+(G)$ is the group of analytic group homomorphisms from G to $\mathbb{C}$ (the additive characters of G).

An algebraic group is an affine algebraic group over $\mathbb{C}$, and usually connected.
A torus is an algebraic group isomorphic to a product of copies of $\mathrm{GL}_1\mathbb{C}$.

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The affine coordinate ring of the algebraic group G is denoted $\mathbb{C}[G]$.

If V is an affine variety over $\mathbb{C}$, $\mathbb{C}[V]$ is its algebra of polynomial functions.

If G acts on V algebraically, $\mathbb{C}[V]^G$ denotes the G-invariant polynomial functions on V.

If L is a Lie algebra, $Z(L)$ denotes the center of L.

If $x, y \in L$, $\mathrm{ad}(x)(y) = [x, y]$.

If $L = \mathrm{Lie}(G)$, the adjoint representation of G on L is denoted Ad, and if H is a subgroup of G, $L^H = \{x \in L \mid \mathrm{Ad}(h)(x) = x$ for all $h \in H\}$.

If X is a group and H, K are subgroups of G, then (H, K) is the subgroup generated by commutators $hkh^{-1}k^{-1}$ with $h \in H$ and $k \in K$.

$X^{ab} = X/(X, X)$.

$\mathbb{C}[X]$ denotes the complex group ring of X.

e is the identity of X.

If G, N are groups and $s: G \to \mathrm{Aut}(N)$ is a homomorphism, the resulting semidirect product group is denoted $N \times_s G$.

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