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978-0-521-07022-5 - The Theory of Economic Policy: Statics and Dynamics

A. J. Preston and A. R. Pagan

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Statics and dynamics

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Preface

This book has as its basic objective a unified treatment of that area of economic theory now commonly known as, following the pioneering work of Tinbergen and Hansen in the 1950s, *the theory of economic policy*. It is a theoretical rather than an applied or econometric study. On the crucial assumptions of a certainly known and linear economic structure, the book focuses on the abstract theory of policy implied by a generic linear deterministic policy problem. This generic policy problem is defined as resulting from the interaction of a policy objective, representing some abstract policymaker's desires, with a policy model, representing the feasible outcomes of policy actions. By identifying a variety both of policy objectives and of policy models within the imposed limitations of certainty and linearity, it is possible to encompass and thus unify a variety of policy problems in a common perspective.

The procedure for attaining this perspective is to begin from the known territory of the static theory of policy. In the work of Tinbergen, Hansen and Theil, two broad policy objectives – the fixed objective and the flexible objective – are associated with a given linear static policy model. In some relevant region of the target space, the fixed objective specifies a particular configuration of targets as desired; whereas the flexible objective alternatively specifies a preference ordering over all target configurations in that region. By assumption, the policy model is a linear mapping from the instrument space to the target space; while the fixed and flexible objectives specify some additional information about the target space.

For either of these policy objectives, the book identifies three major conceptual issues: (i) the *existence* of a policy achieving the stipulated objective; (ii) the *uniqueness* of policy if existing; and (iii) the *design* of policy, whether unique or otherwise. Tinbergen's seminal contribution is interpreted as recognising that these three issues for the fixed objective problem are mathematically equivalent to the existence, uniqueness, and computation of solutions to linear equation systems. For example, as is well known, the Tinbergen counting rule requiring as many instruments as there are targets is simply a reflection of the condition requiring a linear system of equations to have as many solution variables as equations, if it is invariably to possess a solution.

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But the theory of policy presently fails to utilise this existence, uniqueness and design correspondence to its full power. Thus the first task of the book is to formalise the analysis of the fixed objective problem as a problem in the theory of linear mappings – which is simply the theory of linear equations and their solutions in modern dress – and so to utilise this theory to provide a comprehensive treatment of the existence, uniqueness, and design trio. For example, although the conventional formulation runs into difficulties whenever the number of instruments is either smaller than or greater than the number of targets, these difficulties will be shown to be engendered by an inadequate analytic methodology, rather than being intrinsic to the fixed objective problem itself.

The flexible objective problem, with its explicit emphasis on optimisation, is an apparently quite dissimilar type of problem. Yet under the conventional assumption of quadratic preferences, the first-order conditions are themselves a system of linear equations in which the same trio of issues arises. Hence the second major task of the book is to place the linear/quadratic version of the flexible objective problem in as close a correspondence to the fixed objective problem as possible, utilising the linearity of the first-order conditions to do so. The approach adopted here will be familiar from elementary econometric theory dealing with linear least squares theory, in which theory the optimal linear/quadratic solution is viewed as the ‘best’ inexact solution of the linear equation system.

In summary, the static theory of policy will be conceived of as the interaction of either of two types of policy objective – the fixed or the flexible objective – with a linear policy model. Policy existence, policy uniqueness, and policy design then become the central trio of analytical issues; with the theory of linear mappings the appropriate analytic methodology for their investigation.

This re-exploration of the static theory of policy is of value not only in its own right but also in the provision of a conceptual perspective for the analysis of a dynamic theory of policy. This analysis is the third, and by far the most extensive, task of the book. In conducting an analysis of the dynamic theory of policy, our underlying premise will be that in the careful analysis of the fixed and flexible *static* problems much of the content of a *dynamic* theory of policy is already implied.

In the dynamic context, the interaction of the policy objective with the policy model generates a much richer theoretical harvest – because both the objective and the model have structures more complicated dynamically than their static counterparts. Various features emphasise the beginnings, however, of a theoretical unity of the static and dynamic theories. As for the static theory, dynamic policy objectives are classified into fixed and flexible objectives. After more elaborate manipulations in consequence of the dynamic structure, it is demonstrated that the interaction of either type of dynamic objective with an appropriately formulated linear dynamic model

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generates linear mappings with obvious similarities to the analogous static problems. Hence the linear mapping theory so painstakingly explicated for, and applied to, the fixed and flexible static problems proves also of fundamental relevance to the dynamic theory of policy. Policy existence, policy uniqueness, and policy design issues, augmented by stability considerations, therefore emerge as central issues in the dynamic theory of policy.

In the current state of the theory, this emphasis on the formal similarities of the static and dynamic theories of policy is more than a unifying device. Relative to the static theory of policy, the dynamic theory of policy is considerably underdeveloped; the overwhelming effort so far has been devoted to the enunciation and exploration of the dynamic flexible objective linear/quadratic problem. This means there are substantial gaps both in the received structure of the dynamic theory of policy, especially in the analysis of dynamic fixed objectives, and in the analysis of the interrelationships amongst the components of the dynamic theory. The fourth task to be singled out, therefore, is the elaboration of the fixed objective dynamic theory of policy and of the interrelationships between the dynamic fixed and flexible objective theories.

There is no denying that the progression from statics to dynamics entails a very significant increase in analytical complexity, richness, and detail. Yet this necessary proliferation ought not to be allowed to obscure the conceptual unity shared by the static and dynamic theories of policy. One consequence of an appreciation of this unity is that the seminal contributions made by Tinbergen, Hansen, and Theil are more clearly seen to contain within them the seeds of later advances.

A division of the book into three parts reflects the distinctions between, firstly, statics and dynamics and, secondly, fixed and flexible objectives. Part I is devoted to the static theory of policy; Parts II and III respectively to the analysis of fixed and flexible objectives in the dynamic theory of policy. An overview of the chapters will be found in Section 1.11 following an elementary exposition of the static policy framework.

These objectives and the approach just outlined largely determine the mathematical techniques to be employed. The basic requirement is matrix algebra: for example, representation of equation systems via matrix/vector notation; analysis of the existence and uniqueness of solutions to such systems via rank criteria; and computation of solutions via matrix (regular- or pseudo-) inversion. For adequate analysis of these issues slightly deeper aspects of linear algebra are needed, particularly the idea that both the instrument space and the target space can be decomposed orthogonally and that the issues of policy existence, uniqueness and design can then be dissected in terms of appropriately chosen orthogonal decompositions of the two spaces. Exposure to these aspects of the solution of linear equations is already provided by the standard

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econometric treatment of the linear least squares problem; and the book complements this with a fairly self-contained theoretical exposition in Chapters 2 and 3. No additional linear algebra is subsequently introduced in the book which goes beyond the material of these chapters.

While this linear algebra is by far the major mathematical requirement, two other requirements exist. Firstly, some basic calculus connected with optimisation and involving computation of first-order conditions is needed at a few points. More importantly, the dynamic framework utilises matrix difference equation systems. The difference equation context is deliberately chosen in favour of a differential equation context because of the analytical simplicity of the former. Solutions of matrix difference equations are readily found by iterative substitution; and the requisite manipulations just involve matrix algebra. Well-known stability conditions in terms of characteristic roots and the Cayley–Hamilton theorem are additional mathematical concepts necessitated by the dynamic framework.

The preface is a traditional place of reckoning in relation to the inputs into a book's production function. Both authors were doctoral students of A. W. Phillips: each is conscious of the continuing influence of his humanity and practicality. To him, this book is dedicated in spirit, although we surmise its letter may have elicited his wry and perplexed endorsement. On a wider canvas, the Australian National University has provided both authors with the necessary academic environment and resources. We are here especially indebted to Professors J. D. Pitchford, S. J. Turnovsky, and R. D. Terrell for their assistance at propitious moments. The first author also gratefully acknowledges the generosity of Professor J. E. Richardson, the Australian Commonwealth Ombudsman, a generosity that has enabled the book's completion. To E. Sieper he owes both a professional and personal thanks, all too inadequately reflected in Chapters 8 and 13. Finally, to Debbie Sims, Carolyn Gowing, Joyce Radley, Helmi Patrikka and Diane Hewson go our heartfelt thanks for their expert typing of an unrewarding manuscript. As to the output, the preface is no place of reckoning: for that, the marketplace will be the final arbiter. No economist could say otherwise.

Alan Preston
Adrian Pagan

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