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## Index of Logical Systems

### NOMENCLATURE

The letter **G** stands for **sequent systems** (Gentzen systems):

**G0**-systems have **independent contexts** in two-premiss rules.

**G3**-systems have **shared contexts** in two-premiss rules.

**p** stands for the **propositional** part of a system.

**i** stands for an **intuitionistic** system.

**c** stands for a **classical** system.

**m** stands for a **multisuccedent** system.

**G4ip** is obtained from **G3ip** by changing the left implication rules.

**Extension** of a system by *Rule* is indicated through writing *+Rule*.

A superscript \* denotes an arbitrary extension.

The letter **N** indicates **natural deduction** style:

**GN** is a sequent system in natural deduction style.

**GM** is the corresponding multisuccedent system.

**NG** is natural deduction in sequent style.

**MG** is the corresponding multisuccedent system.

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