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THE FOUNDATIONS OF DIFFERENTIAL GEOMETRY

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BY

OSWALD VEBLEN

AND

J. H. C. WHITEHEAD

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PREFACE

THIS is intended as a companion to the Cambridge Tract No. 24, on Invariants of Quadratic Differential Forms. As its name implies it contains a set of axioms for differential geometry and develops their consequences up to a point where a more advanced book might reasonably begin. Formulae appear only incidentally and the reader is supposed to obtain those needed from the tract No. 24, or from other books and articles on the formal side of the subject.

Analytical operations with coordinate systems are continually used in differential geometry, a typical process being to "choose a coordinate system such that...." It is therefore natural to state the axioms in terms of an undefined class of "allowable" coordinate systems, and to deduce the properties of the space from the nature of the transformations of coordinates permitted by the axioms.

The axioms for differential geometry in general are preceded by more special sets of axioms in which the structure of a space is defined by an appropriate class of "preferred" coordinate systems. Thus Euclidean geometry is characterized by the class of rectangular cartesian coordinate systems. The "preferred" coordinate systems constitute a sub-class of the "allowable" coordinate systems for any one of these spaces. The former class is small, so as to characterize the structure of the space, and the latter is large, so as to permit freedom of analytic operation.

These earlier axioms are found to be adequate for the differential geometry of an open simply connected space, the most elementary theorems of which occupy the greater part of Chaps. III–V. The more general axioms, in terms of allowable coordinate systems and without restrictions on the connectivity of the space, are given in Chap. VI. We believe that they provide an adequate foundation for any of the differential geometries which are now being studied. The complete theory which should be constructed out of these axioms would be a combination of infinitesimal geometry and analysis situs. In the final chapter we outline some of the questions which arise, in the hope that some of the readers of this tract may participate in the construction of a branch of mathematics which we are convinced is of great importance.

O. V.
J. H. C. W.

PRINCETON, N.J.

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