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978-0-521-06447-7 - Topics in the Constructive Theory of Countable Markov Chains

G. Fayolle, V. A. Malyshev and M. V. Menshikov

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[More information](#)

Contents

| | |
|--|-----------------|
| <i>Errata</i> | <i>page</i> vii |
| <i>Introduction and history</i> | 1 |
| 1 Preliminaries | 5 |
| 1.1 Irreducibility and aperiodicity | 6 |
| 1.2 Classification | 8 |
| 1.3 Continuous time | 9 |
| 1.4 Classical examples | 11 |
| 1.4.1 Doeblin's condition | 11 |
| 1.4.2 Birth and death process | 12 |
| 1.4.3 The space homogeneous random walk on \mathbf{Z}^m | 13 |
| 2 General criteria | 16 |
| 2.1 Criteria involving semi-martingales | 16 |
| 2.2 Criteria for countable Markov chains | 26 |
| 3 Explicit construction of Lyapounov functions | 33 |
| 3.1 Markov chains in a half-strip | 33 |
| 3.1.1 Generalizations and problems | 35 |
| 3.2 Random walks in \mathbf{Z}_+^N : main definitions and interpretation | 37 |
| 3.3 Classification of random walks in \mathbf{Z}_+^2 | 39 |
| 3.4 Zero drifts | 56 |
| 3.5 Jackson networks | 62 |
| 3.6 Asymptotically small drifts | 72 |
| 3.7 Stability and invariance principle | 76 |
| 4 Ideology of induced chains | 79 |
| 4.1 Second vector field | 79 |
| 4.2 Classification of paths | 82 |
| 4.3 Gluing Lyapounov functions together | 86 |
| 4.4 Classification in \mathbf{Z}_+^3 | 92 |

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G. Fayolle, V. A. Malyshev and M. V. Menshikov

Frontmatter

[More information](#)*Contents*

| | | |
|----------|---|-----|
| 5 | Random walks in two-dimensional complexes | 98 |
| 5.1 | Introduction and preliminary results | 98 |
| 5.2 | Random walks on hedgehogs | 103 |
| 5.3 | Formulation of the main result | 104 |
| 5.4 | Quasi-deterministic process | 108 |
| 5.5 | Proof of the ergodicity in theorem 5.3.4 | 111 |
| 5.6 | Proof of the transience | 114 |
| 5.7 | Proof of the recurrence | 118 |
| 5.8 | Proof of the non-ergodicity | 121 |
| 5.9 | Queueing applications | 123 |
| 5.10 | Remarks and problems | 130 |
| 6 | Stability | 131 |
| 6.1 | A necessary and sufficient condition for continuity | 131 |
| 6.2 | Continuity of stationary probabilities | 137 |
| 6.3 | Continuity of random walks in \mathbf{Z}_+^N | 144 |
| 7 | Exponential convergence and analyticity | 148 |
| 7.1 | Analytic Lyapounov families | 148 |
| 7.2 | Proof of the exponential convergence | 150 |
| 7.3 | General analyticity theorem | 157 |
| 7.4 | Proof of analyticity completed | 161 |
| 7.5 | Examples of analyticity | 163 |
| | <i>Bibliography</i> | 165 |
| | <i>Index</i> | 168 |

Cambridge University Press

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G. Fayolle, V. A. Malyshev and M. V. Menshikov

Frontmatter

[More information](#)**List of errata**

– **Page 8, Theorem 1.2.1** should be modified as follows:

Theorem 1.2.1 *If $Q(\alpha, \alpha) = 1$ for some α , then $Q(\beta, \beta) = 1$ for all β . Similarly, if $m_{\alpha\beta} + m_{\beta\alpha} = \infty$ for some pair (α, β) , then $m_{\alpha\beta} + m_{\beta\alpha} = \infty$ for all (α, β) .* ■

– **Page 8, Definition 1.2.2** should be modified as follows:

Definition 1.2.2 *An irreducible aperiodic MC is called*

- (i) recurrent if $Q(\alpha, \alpha) = 1$, at least for one state α ;
- (ii) non recurrent or transient if $Q(\alpha, \beta) < 1$, for some pair (α, β) ;
- (iii) positive recurrent or ergodic, if $m_{\alpha\beta} + m_{\beta\alpha} < \infty$, at least for one pair (α, β) ;
- (iv) null recurrent if $Q(\alpha, \alpha) = 1$ and $m_{\alpha\alpha} = \infty$, at least for one α ;
- (v) non ergodic if $m_{\alpha\beta} = \infty$, at least for one pair (α, β) .

– **Page 10, equation (1.9)** should be :

$$P(\xi_0 = \alpha_0, \dots, \xi_{t_n} = \alpha_n) = p_{\alpha_0}(0)p_{\alpha_0\alpha_1}(t_1) \dots p_{\alpha_{n-1}\alpha_n}(t_n - t_{n-1}).$$

– **Page 17, the left member of equation (2.3)** should be $E(\tilde{Y}_{n+1}/\mathcal{F}_{\tilde{N}_n})$.

– **Page 21, two lines above inequality (2.20) :**

replace $E(|S_n - S_{n+1}|^\alpha / \mathcal{F}_n)$ by $E(|\tilde{S}_n - \tilde{S}_{n+1}|^\alpha / \mathcal{F}_n)$.

– **Page 28, line 6 from bottom, write**

$$f(\alpha_j) = P\{\xi_\nu = \alpha_0, \xi_k \neq \alpha_0, 1 \leq k < \nu / \xi_0 = \alpha_j\}, j \geq 1.$$

– **Page 41, in the statement of theorem 3.3.2** replace “conditions A and B hold.” by “condition A holds.”

– **Page 56, lines 2 and 8 from above, write $\frac{1}{2} \leq \delta \leq 1$ instead of $\frac{1}{2} < \delta \leq 1$.**

– **Page 56, line 8 from below: replace “Condition B” by “Condition C”.**

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Frontmatter

[More information](#)

– **Page 59, one line before equation (3.51):** “Condition B” should be “Condition C”.

– **Page 64:** replace equation (3.60) by the following system (3.60):

$$\begin{cases} q_{\alpha\beta} = w_{\alpha} \lambda_{\alpha\beta}, & \alpha \neq \beta, \\ q_{\alpha\alpha} = 1 - \sum_{\alpha \neq \beta} q_{\alpha\beta}. \end{cases}$$

– **Page 64, last line :** write “ $q_{\alpha\beta} = 0$ ” instead of “ $p_{\alpha\beta} = 0$ ”.

– **Page 68, 2 line from below:** should be

$$\Gamma_{\wedge}^{\alpha} \cap \overline{B^{\wedge'}} = \emptyset, \quad \text{for } \wedge' \subseteq \wedge,$$

– **Page 72, in the statement of lemma 3.5.10,** replace “ $i \in 1, \dots, N$ ” by “ $i \in \{1, \dots, N\}$ ”

– **Page 73, equation (3.79) should be**

$$\mu_1(x) \leq -\frac{\theta \mu_2(x)}{2x}, \quad \text{for } x \geq B,$$

– **Page 76,** the first equation of theorem 3.6.4 should be

$$\mathbf{v}(\mathbf{x}) = \mathbf{v} + O(\|\mathbf{x}\|^{-\delta}), \quad \text{for some } \delta > 0,$$

– **Page 78, in (ii) of theorem 3.7.4,** replace “nullrecurrent” by “null recurrent”

– **Page 82,** line 6 from above: replace “ $\eta_0 > 0$ ” by “ $n_0 > 0$ ” .

– **Page 83,** lines 3 and 4 from above (under the Figures) : replace “chain” by “face”.

– **Page 86,** insert a dot at the end of Equation (4.4) and start the next sentence by “We” instead of “we”.

– **Page 88, third line in the proof of lemma 4.3.3:** replace

$$“\alpha \in \mathbf{Z}_+^N \setminus B_{R_1 R_2}^{\wedge}” \quad \text{by} \quad “\alpha \in \mathbf{Z}_+^N \cap B_{R_1 R_2}^{\wedge}”.$$

– **Page 92,** 10 lines from below: the formula should be

$$p_{\alpha\beta} = 0, \quad \text{for } (\beta_i - \alpha_i) > -d, \quad \forall i = 1, \dots, N,$$