

Cambridge University Press

978-0-521-06439-2 - Affine Differential Geometry: Geometry of Affine Immersions

Katsumi Nomizu and Takeshi Sasaki

Table of Contents

[More information](#)

CONTENTS

Preface	vii
Introduction	ix
Chapter I Affine geometry and affine connections	
1. Plane curves	1
2. Affine space	7
3. Affine connections	11
4. Nondegenerate metrics	18
5. Vector bundles	22
Chapter II Geometry of affine immersions – the basic theory	
1. Affine immersions	27
2. Fundamental equations. Examples	32
3. Blaschke immersions – the classical theory	40
4. Cubic forms	50
5. Conormal maps	57
6. Laplacian for the affine metric	64
7. Lelievre’s formula	68
8. Fundamental theorem	73
9. Some more formulas	77
10. Laplacian of the Pick invariant	82
11. Behavior of the cubic form on surfaces	87
Chapter III Models with remarkable properties	
1. Ruled affine spheres	91
2. Some more homogeneous surfaces	95
3. Classification of equiaffinely homogeneous surfaces	102
4. $SL(n, \mathbf{R})$ and $SL(n, \mathbf{R})/SO(n)$	106
5. Affine spheres with affine metric of constant curvature	113
6. Cayley surfaces	119

Cambridge University Press

978-0-521-06439-2 - Affine Differential Geometry: Geometry of Affine Immersions

Katsumi Nomizu and Takeshi Sasaki

Table of Contents

[More information](#)

vi

Contents

7. Convexity, ovaloids, ellipsoids	122
8. Other characterizations of ellipsoids	125
9. Minkowski integral formulas and applications	129
10. The Blaschke–Schneider theorem	138
11. Affine minimal hypersurfaces and paraboloids	141

Chapter IV Affine-geometric structures

1. Hypersurfaces with parallel nullity	147
2. Affine immersions $\mathbf{R}^n \rightarrow \mathbf{R}^{n+1}$	152
3. The Cartan–Norden theorem	158
4. Affine locally symmetric hypersurfaces	161
5. Rigidity theorem of Cohn–Vossen type	165
6. Extensions of the Pick–Berwald theorem	169
7. Projective structures and projective immersions	174
8. Hypersurfaces in \mathbf{P}^{n+1} and their invariants	181
9. Complex affine geometry	187

Notes

1. Affine immersions of general codimension	196
2. Surfaces in \mathbf{R}^4	198
3. Affine normal mappings	202
4. Affine Weierstrass formula	203
5. Affine Bäcklund transformations	209
6. Formula for a variation of ovaloid with fixed enclosed volume	213
7. Completeness and hyperbolic affine hyperspheres	215
8. Locally symmetric surfaces	217
9. Centro-affine immersions of codimension 2	221
10. Projective minimal surfaces in \mathbf{P}^3	230
11. Projectively homogeneous surfaces in \mathbf{P}^3	230

Appendices

1. Torsion, Ricci tensor, and projective invariants	235
2. Metric, volume, divergence, Laplacian	240
3. Change of immersions and transversal vector fields	242
4. Blaschke immersions into a general ambient manifold	243

Bibliography	246
------------------------	-----

List of symbols	256
---------------------------	-----

Index	258
-----------------	-----